Globally optimized finite-difference extrapolator for strongly VTI media

Jin-Hai Zhang[1] and Zhen-Xing Yao[1]

I - (FD) . T T FD T - 0.1%. F 0.5% . T

(E , 1994). I (FD) 2008; B , 2009;
(C , 1985); , FD
. F , FD
. T ,
FD -
VTI ,
- (R , 1999; H
, 2005; ., 2005 ; H ., 2006; F L ,

T . A
, FD .
A VTI S (2009), T
. O
- FD
58 ;
36 . F - FD , 77
53 .

T 2D VTI
(A , 1998, 2000; S ,
2009)

$$\frac{\partial P}{\partial z} = ik_z P, \tag{1}$$

k_z

$$k_z = \frac{}{v}\sqrt{1 - \frac{(1+2\delta)u^2}{1-2(\varepsilon-\delta)u^2}}, \tag{2}$$

$v \equiv v(x,z)$ qP-
, ε δ T (T ,
1986), $u^2 \equiv v^2 k_x^2/$ ^{2}, , $i = \sqrt{-1}$
, $P \equiv P(x,z;\ )$
. T k_z N -
T :

$$k_z \approx \frac{}{v}\sum_{n=0}^{N} t_n u^{2n}, \tag{3}$$

$$\begin{aligned} t_0 &= 1, \quad t_1 = -\frac{1}{2}\ , \quad t_2 = -\frac{1}{8}(\ ^2 + 4\ \ ), \\ t_3 &= -\frac{1}{16}(\ ^3 + 4\ ^2\ + 8\ \ ^2), \\ t_4 &= -\frac{1}{128}(5\ ^4 + 24\ ^3\ + 48\ ^2\ ^2 + 64\ \ ^3), \end{aligned} \tag{4}$$

$\equiv (1+2\delta)$ $\equiv 2(\varepsilon-\delta)$.
E 3 - FD u
(M , 1982; C ,
1985)

$$k_z \approx \bar{k}_z = \frac{}{v}\left(1 - \frac{\ _1 u^2}{1 -\ _1 u^2} - \frac{\ _2 u^2}{1 -\ _2 u^2} - \blacktriangle\blacktriangle\blacktriangle\right). \tag{5}$$

U (M , 1982;
R R , 1994; F L , 2008),
5 (. ., $_1$, $_1$, $_2$, $_2$, $\blacktriangle\blacktriangle\blacktriangle$)
$t_1 \sim t_N$. F - FD (. ., $N = 2$),
$_1 = 0.5$ $_1 = 0.25\ +$. F - FD
(. ., $N = 4$), $_1 = 0.361803340$, $_1 = 3.05572809\ +$,
$_2 = 0.13819660$, $_2 = 2.61803399\ +$.
A $\partial^2/\partial x^2 \to -k_x^2$, - FD

$$\frac{\partial P}{\partial z} = \frac{i}{v}P, \tag{6}$$

$$\frac{\partial P}{\partial z} = \frac{i\ _1 \frac{v}{}\frac{\partial^2}{\partial x^2}}{1 +\ _1 \frac{v^2}{^2}\frac{\partial^2}{\partial x^2}}P, \tag{7}$$

$$\frac{\partial P}{\partial z} = \frac{i\ _2 \frac{v}{}\frac{\partial^2}{\partial x^2}}{1 +\ _2 \frac{v^2}{^2}\frac{\partial^2}{\partial x^2}}P. \tag{8}$$

T - FD 6 7.

5
FD .
FD

$$E \equiv \left|\frac{\bar{k}_z - k_z}{k_z}\right|. \tag{9}$$

T (S , 2009)

$$\sin^2 \ = \frac{V^2(\ )k_x^2}{^2}, \tag{10}$$

$$\begin{aligned} \frac{V^2(\ )}{v^2} &= \frac{1}{2} + \varepsilon\sin^2 \\ &\quad + \frac{1}{2}\sqrt{(1+2\varepsilon\sin^2\ )^2 - 2(\varepsilon-\delta)\sin^2 2\ }, \end{aligned} \tag{11}$$

, $V(\ )$ qP- VTI v
(T , 1996; A ,
1998, 2000). R $u^2 \equiv v^2 k_x^2/$ ^{2},

$$u^2 = \frac{v^2\sin^2\ }{V^2(\ )}. \tag{12}$$

T , (. ., 9)
T -
I ε δ.
, ε δ
T $\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]$

$\delta \in [\delta_{\min}, \delta_{\max}]$;

$\in [0\ ,\ _{\max}]$

T (.., 1% 0.5%). I

ε δ

$\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]$ $\delta \in [\delta_{\min}, \delta_{\max}]$,

$\varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}]$ $\delta \in [\delta_{\min}, \delta_{\max}]$. I

T

$E(\ ,\varepsilon,\delta) \le T$. N

max.

T

$\bar{k}_z$ (5)

k_z (2) . D

(., 2008). T ,

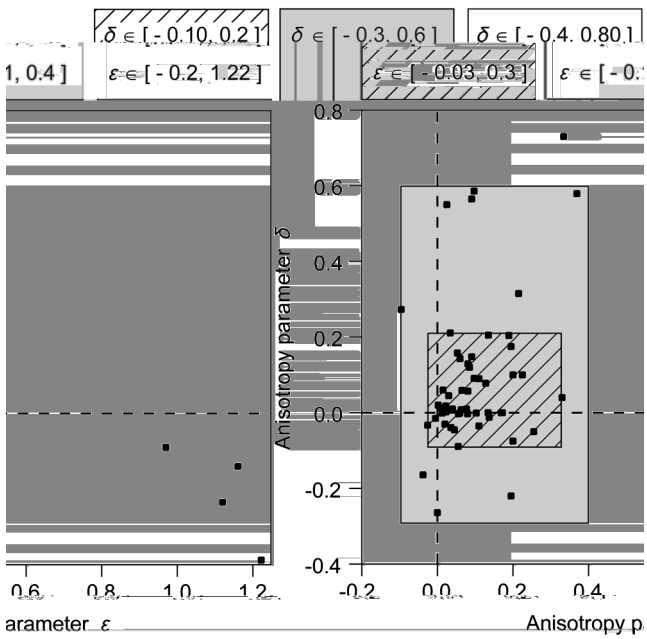
- FD

- FD . B -

(. ., 6) . T

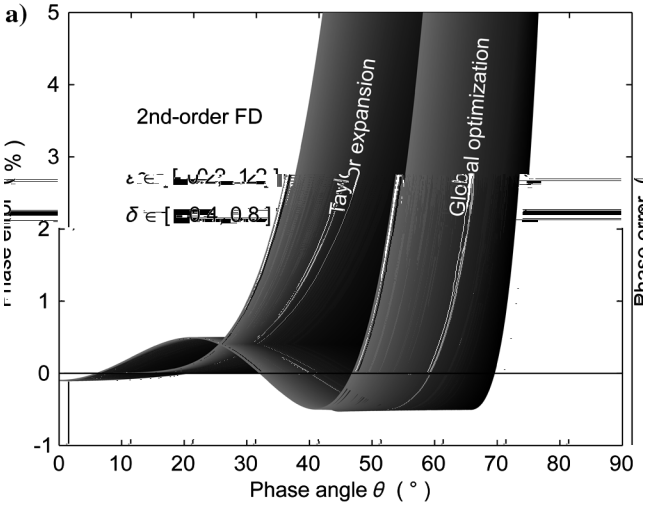
ε δ ; , - ε δ. T - ε δ . C - ε δ, . I , . F - 0.001. U F. , 1. F. , 1 T - (. ., 5), FD - (. ., 14) FD (. ., 13), ( ) . F 0.001. F. , 1 ε = 0.4 δ = 0.2, F. , 1 ε = 0.2 δ = −0.2. F , . I FD T - 45 . M . I T - H 14 1% (S , 2009) ; , . S 1% 63 . T 14 . A 0.0005, - I 13 0.5% . T . T (5) 14 . H 0.5% 1%. T 13

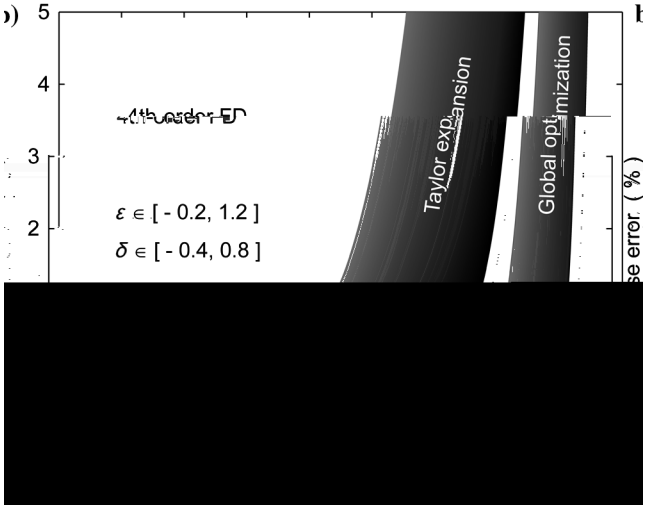
14 , - ε δ T (1986), F. , 2. T : ε ∈ [−0.03, 0.3] δ ∈ [−0.1, 0.2] (F. , 2), ε ∈ [−0.1, 0.4] δ ∈ [−0.3, 0.6] (F. , 2), ε ∈ [−0.4, 0.8] δ ∈ [−0.2, 1.22] (F. , 2). O . T δ ε, .T . A ε ∈ [ε_min, ε_max] δ ∈ [δ_min, δ_max] - 0.02. F. , 3, 4, 5 T -



F. , 2. S T - . T T 1 T (1986).

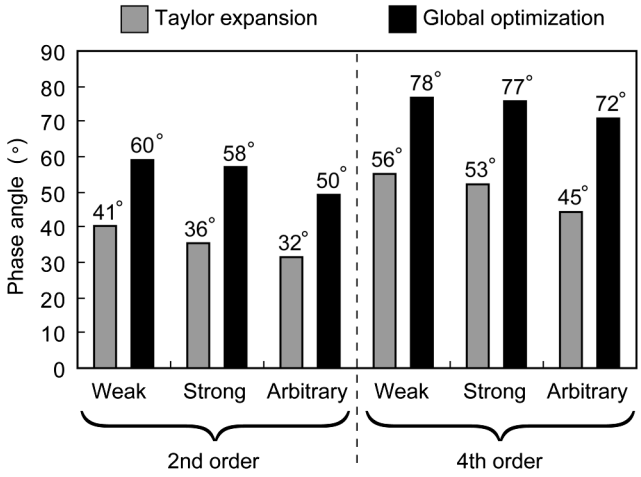
I . A 2D 2000×600 5 . T qP- $v = 4500$ m/s. A . T 600 2 . T R 30 H . F. 7 7 - FD , . T $\varepsilon = 0.4$ $\delta = 0.6$ (. ., S4 F. 8), . E T : - . O , FD - T . F. 7



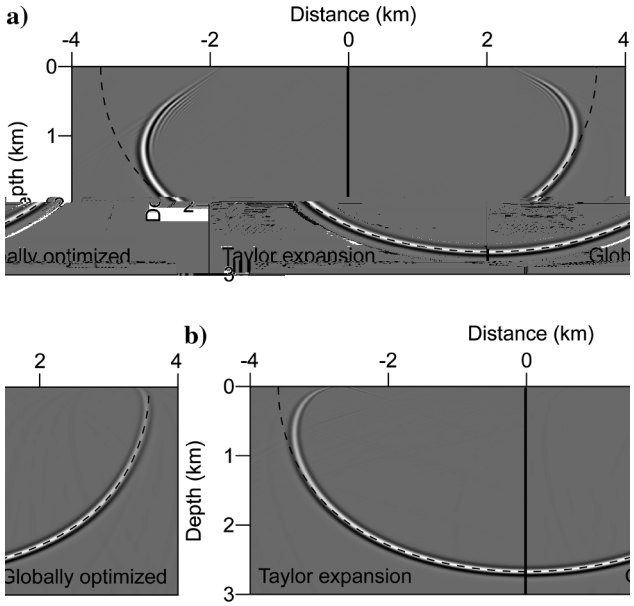


F. 5. R : ( ) FD , ( ) - FD . S F. 3

FD . F. 8 T F. 2. F. 9, 10, 11 - FD -

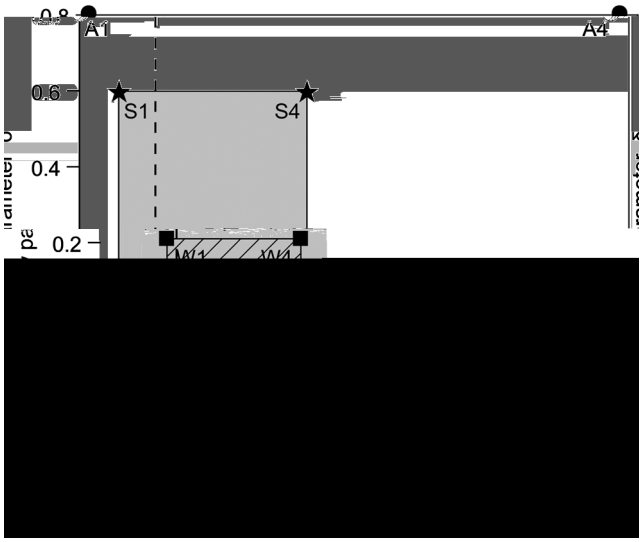


F. 6. T F. - 3, 4, 5. T . T T - FD FD .

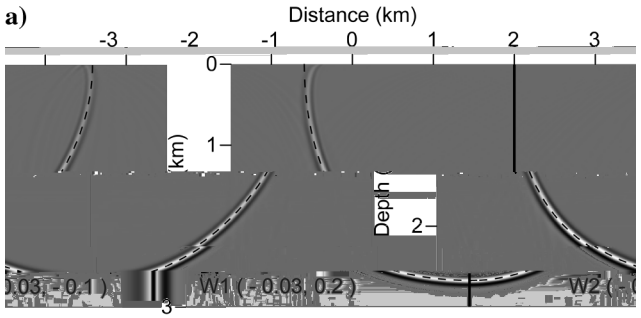


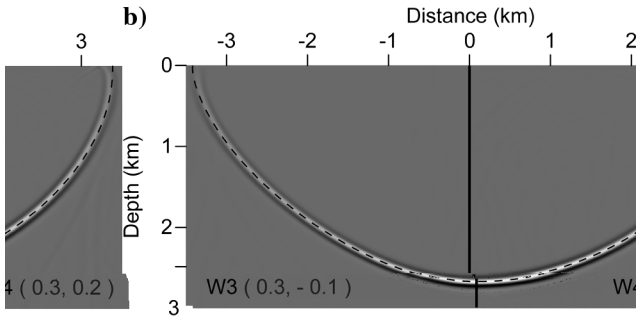
F. 7. I . T $\varepsilon = 0.4$ $\delta = 0.6$. ( ) T - FD . ( ) T - FD . E : T - FD ,

( ), . T
FD
VTI .



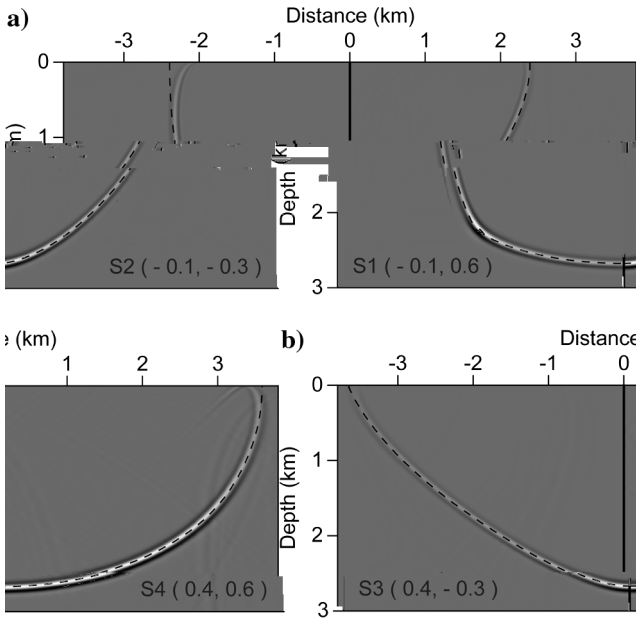
F. , 8. S
F. , 9, 10, 11. T . , F. , 2, . F , S , , A .



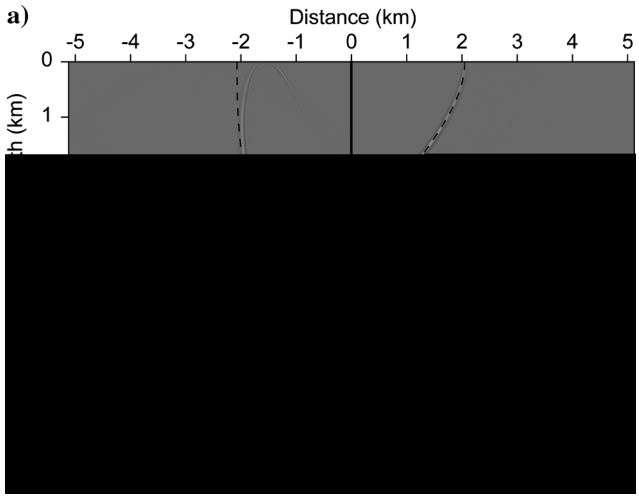


F. , 9. I , FD . E , , (., 1– 4 F. , 8). T .

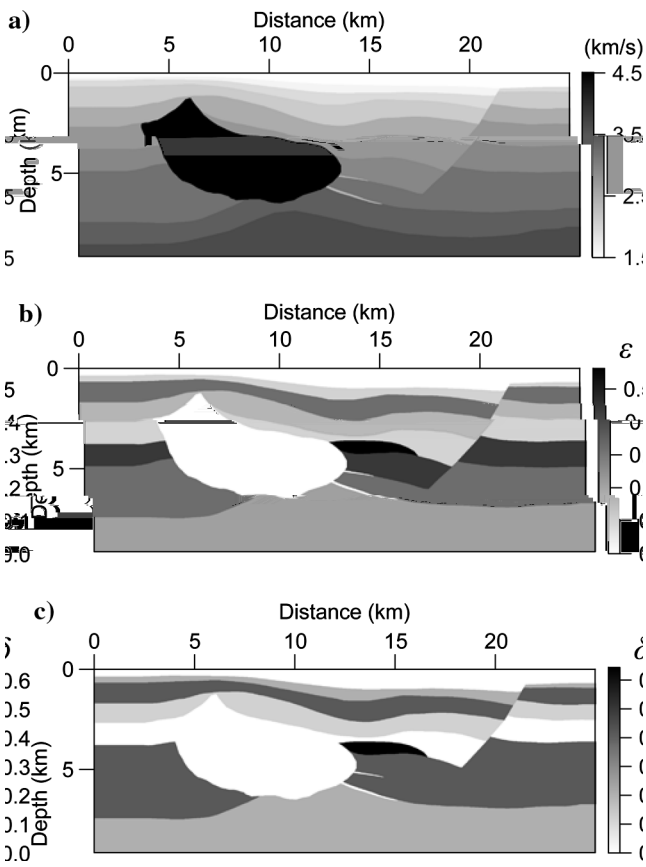
T FD , H , F. , 12. T T ε δ , . T 2048 × 750 40 . A



F. , 10. I , FD . E , , (., S1–S4 F. , 8). T .



F. , 11. I , FD . E , , (., A1–A4 F. , 8). T .



F. , 12. S H : ( ) *q*P- , ( ) $\varepsilon = \in [0, 0.57]$, ( ) $\delta \in [0, 0.63]$. T H : ε δ ,

F. , 13. C - FD : ( ) , T - , ( ) ,

. M

T VTI

*q*S-

. H

. C

. O

3D

(.., L , 1991; . , 2001; F E , 2002; B , 2009; . , 2009; , 2011).

FD T

H , B. L., H. C , P. , 2006, 3D ,
F , -
: 76 A , I M . , SEG, E A , 2387–
2391.
H , L. J., M. C. F , 2000, G F , - -
: 70 A , I M . , SEG, E -
A , 802–805.
K , S., C. D. G , M. P. V , 1983, O -
, , : S , , 671–680, : 10.1126/ .220
.4598.671.
L , K., J. K. C , 1993, M
: G , , 1454–1467,
: 10.1190/1.1443360.
L , M., S. S , 1985, O - , : G -
, , 1634–1637, : 10.1190/1.1441853.
L R , , J. H., 1997, D - - , - , : 67
A , I M . , SEG, E A , 1703–1706.
L R , , J. H., M. V. H , 2001, S -
: G , , 1538–1550, : 10.1190/1.1487100.
L , ., 1991, C - 3D
: G , 1650–1660, : 10.1190/1.1442975.
L , L., J. . , 2006, O , - F , -
R L , , L09308, : 10.1029/2006GL025849.
M , ., 1982, S : O G
P (C), , 6–15.
M ,M. A., . L. A , 1994, 3-D - -
: 64 A , I M . ,
SEG, E A , 1205–1208.
P , ., B. U , H. K. H , 2010, O - - ,
VTI : G , , . 6, S237–S248. : 10.1190/1.3509466.
R , J., C. G , J. M C , M. O , 2005, P
- , VTI : T L E , ,
618–620, : 10.1190/1.1946218.
R , D., 1999, M , : J S E , , 39–55.
R , D., T. R , 1994, F , - : G -
, , 1882–1893, : 10.1190/1.1443575.
S , M. K., P. L. S , 1991, N - -
, , : G , , 1624–1638,
: 10.1190/1.1442973.
S , G., 2009, O - F , -
VTI : G , , . 6, CA189–
CA197. : 10.1190/1.3202306.
T , L., 1986, : G , , 1954–1966,
: 10.1190/1.1442051.
T , J. ., K. , G. S , 2004, D -
, , LSQ
: G , , 1037–1045, : 10.1190/1.1778246.
T , I., 1996, P-
: A : G , , 467–483, : 10.1190/1.1443974.
U , , O., 1995, 2-D
, , : G , , 1819–1829, : 10.1190/1
.1443914.
, ., 2001, ADI , : A , -
3D , : G P , , 547–556,
: 10.1046/ .1365-2478.2001.00278. .
, . B., R. S. , , 1998, I ,
, : 68 A , I M . , SEG,
E A , 1811–1814.
, . B., R. S. , , 1999, I -
: 69 A , I M -
. , SEG, E A , 1863–1866.
. , J., D. J. V , , C. P. A. , 2001, D
, G P , , 287–299, : 10.1046/
.1365-2478.2001.00255. .
. , J. H., . M. , S. Q. , . . , 2010, O
C F , : A - , -
: G , , . 2, S23–S34,
: 10.1190/1.3350861.
. , J. H., . . , 2011, R - FFD
: G , , . 4, S165–S175, : 10
.1190/1.3590214.
. , L., B. H , H. C , 2005 , 3D F , -
: 75 A , I M . , SEG,
E A , 1914–1917.
. , L., J. . R , G. M. H , 2005 , F - -
, , 843–852, : 10.1111/ .2005.53. , -6.
. , ., 2009, F , - ,
VTI : G P -
P , (C), , 27–33.
. , ., H. . , 2009, A TTI
: 79 A , I M . , SEG, E A -
, 2794–2798.
, S. ., J. H. . , . . , 2008, G F ,
, - : C J , G (C), ,
1844–1850.