, 2015, 46, 253–260

h : //d .d i. g/10.1071/EG14016

# Accele a ing ei mic in e ola ion i h a g adien ojec ion me hod ba ed on igh f ame o e of c ele

Jingjie Cao<sup>1,4</sup> Yanfei Wang<sup>2</sup> Benfeng Wang<sup>3</sup>

**Ke od:** c qe a f , g adie jeci eh d, i e e ble ,  $L_1$  eg la i a i , a efiq d

Recei ed 11 Feb. a 2014, acce ed 24 J. e 2014, bji hed hi e 6 A. g. 2014

# In od c ion

Sei ic da a ha i ja e he N i -Sha a ji g he e a b i g ha f j ajia e a d de e i ha e he e l f i g ai , l i jeqi i ai , de-h i i g, a dAVOa a i (Li, 2004, Naghi adeh a d Sacchi, 2010). Sei h ici e l ja i i a ajid ech i e e ha ce a ji g de i , b e i g aia ajia i g a d i h i g i agi g acc ac (S i , 1991; K ei e a d Sacchi, 2013), a d f a c ciaj e i he ei ic h ce i g fl .

Ma i e ha i e h d ha e bee ed i a decade, à d ig a ce i g ba ed e h d a he he ai ea ha ê e (D. ij da e a ., 1999; Li, 2004; Naghi adeh a d Sacchi, 2010; S i , 1991). A i

ba ed a i / e c le i ha e bee ed (Ya g e a, 2012; K ei e a d Sacchi, 2012, 2012, 2013).

A a la ge-ca e c i g he, ei ic da a i e la i e ie efficie e h e h d ed ce he h c ea i g h c a i a c h. Ab a a d Kabi (2006) h d h ced he jec i c e e (POCS) e h d le el ie ic i e h la i . Z a je a d Sacchi (2007) ad ed i e a i e e e e le h e h e h e la i . The i e a i e f h e h l di g (IST) e h d a i d ced b He a a d He e fe (2008). A i e e f he IST e h d, he fa la i e f h e h l di g a la e h e la i a c f h e IST. c f he IST  $\stackrel{\cdot}{e}$ h d. The ec a jec ed g adie f  $\stackrel{\cdot}{L}_1$  i i i a i (SPGL1) e h d ca b ai b a  $e^{\stackrel{\cdot}{h}}_1$  i f  $\stackrel{\cdot}{L}_1$  c  $\stackrel{\cdot}{a}$  i by e (a  $\stackrel{\cdot}{d}$  be  $\stackrel{\cdot}{g}$  a d F ied a de, 200 $\stackrel{\cdot}{g}$ ). The e  $\stackrel{\cdot}{e}$ h d, h e e, a  $\stackrel{\cdot}{e}$  i e c  $\stackrel{\cdot}{h}$  i g f he h ge ei ic da a e, he ef e e efficie  $\stackrel{\cdot}{e}$ h d h 1d be e ea ched.

e ea ched. 
All heab e eh d e he  $L_1$  ea e he a i f l i , b he  $L_1$  i diffe e iable a igi, a d g adie - e eh d ca be a lied di ecl. Re ea che i e d le e he eglai ai ha a e e he ed be ch e ca ef ll b ai a e e l i l l de e c e he ed h ed h a i a e i, he he g adie -ba ed eh d

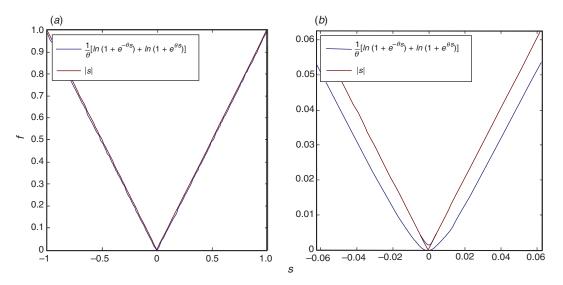
<sup>&</sup>lt;sup>1</sup>Shijiazhuang University of Economics, Shijiazhuang, Hebei 050031, China.

<sup>&</sup>lt;sup>2</sup>Key Laboratory of Petroleum Resources Research, Institute of Geology and Geophysics,

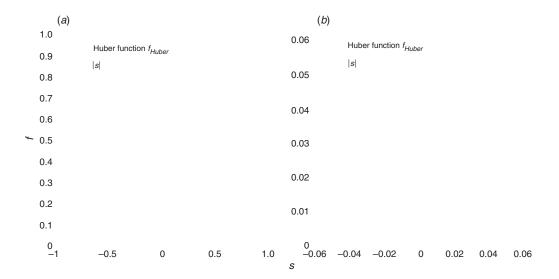
Chinese Academy of Sciences, PO Box 9825, Beijing 100029, China.

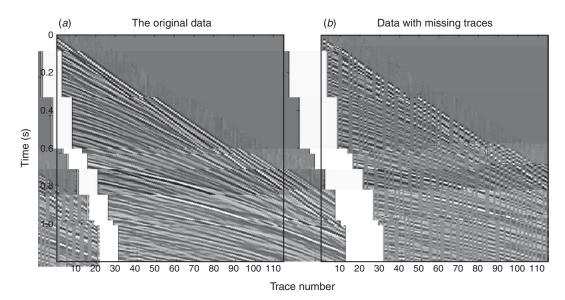
<sup>&</sup>lt;sup>3</sup>State Key Laboratory of Petroleum Resources and Prospecting, China University of Petroleum, Beijing 102249, China.

<sup>&</sup>lt;sup>4</sup>Corresponding author. Email: cao18601861@163.com



 $\mathbf{F}_{\mathbf{z}}^{\mathbf{z}}$  . 2. ( )  $_{\theta}($  ) i h  $\theta$  = 10000. ( ) Mag ified ie f( ).





 $\mathbf{F}_{\mathbf{a}}^{\mathbf{g}}$  . 4. ( ) O  $\mathbf{igi}_{\mathbf{b}}$  a h da a. ( ) Sa 1ed h da a.

Table 1. C = a =  $\frac{1}{1}$  L<sub>1</sub>, FISTA a<sub>1</sub> d SPGL1 = e<sub>1</sub> = d = ... da a.

	S h L <sub>1</sub>	FISTA	SPGL1
CPU i e()	56	73	156
SNR (db)	10.4975	9.8556	9.9523
Rejai e e	0.2986	0.3215	0.3180

ca be a lied a d he eglaiai aa ee i e ied bahed he gadie jeci eh d ih h  $L_1$  h a i ai .

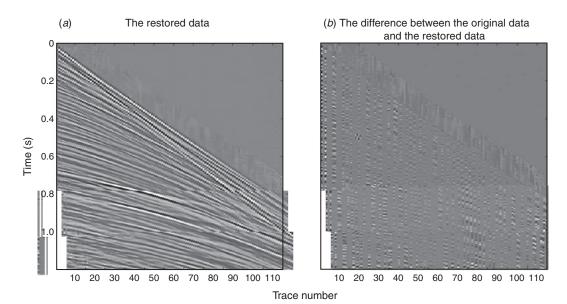
I hi ahe, e h  $L_1$  f ci ae aal ed fi h he he H be f ci ich e ha h iae he  $L_1$  . A called a de fah gadie jeci eh d ih ed le he ealiched he ed eh d ihe ef he igh fa e he f he called a fahed eh dihe e f he igh fa e he f he called a le he icad ealei icada de el icada de el he alidi f he he ed eh d.

# S a e o imi a ion model of ei mic in e ola ion

Mathematical model of seismic sparse interpolation

Sei ic i e la i ca be ea ed a a i e e ble, a d he f  $\overset{h}{a}$ d ble  $\overset{h}{ca}$ a be de ed a  $\overset{h}{n}$ h e

$$\Phi = (1)$$



256 J. Ca e aq.

2013; Wa g e a ., 2013). If  $=\Psi$  i a e, he e  $\Psi$  i a h g  $_{h}$  a f igh fa e, e a i  $_{h}$  1 ca be cha ged  $_{h}$ 

$$\Phi = \Phi \Psi^* = = = 1, \qquad (2)$$

he e  $\Psi^*$  i he He i ia a e f  $\Psi$  a d =  $\Phi\Psi^*$ . Ma e h d ha e bee de q h ed h fi d a e h i e a ih 2, ch a g eed h a g i h (Maha a d Zha g, 1993), c e h i i a i (Bec a d Teb III e, 2009; h d de Be gh a d F ied a de h, 2009; h e a l, 1998) h d h -c h i i h i h i e h d h h h e e h i i g j i i fica i a e i a h e f I a ge-cape c a i (Ca e a l, 2012; h Che e a l, 1998). The c h e d c e i i a i h i h e ba i l b e :

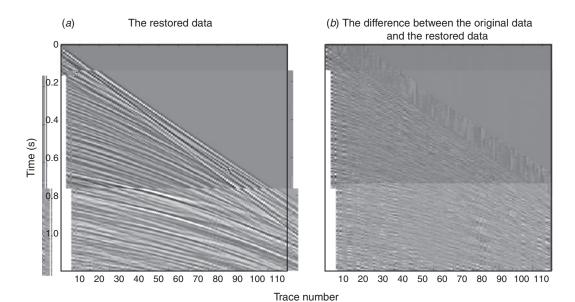
$$\begin{array}{cccc}
i_{\mathbf{L}} \| & \|_{1} & \dots & = 1, \\
\end{array} \tag{3}$$

2005). Beca e he bjeci e f ci f e ai 3 i diffe e iable a igi, i ca be h e d di eci h e c j ga e g adie e h d a d Ne h e e h d di eci h e e a i d e e h e d be h e c j ga e e a che ha e e e h e h e c ai ed f h f e ai 3:

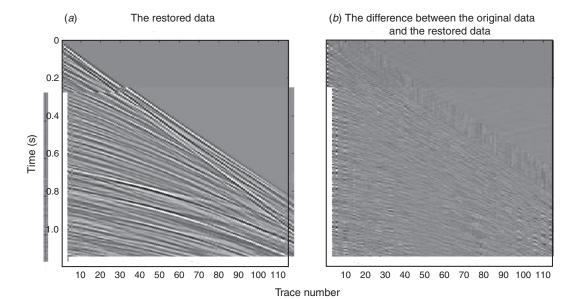
$$\| - \|_{2}^{2} + \lambda \| \|_{1}, \tag{4}$$

 $i_{h} \| - \|_{2}^{2} + \lambda \| \|_{1}, \qquad (4)$   $i_{h} \| - \|_{2}^{2} + \lambda \| \|_{1}, \qquad (4)$   $i_{h} \| f \| e \| \text{ a le he IST a d FISTA ehd, b. he eglaiai, a a ee } \lambda \| h \|_{1} \text{ d be adj. ed ca ef }_{1} \| .$   $A \| h \| h \|_{1} \| e \| e \|_{1} \| e \|_$ 

$$i_{\text{th}} \quad (\ ) \ \ldots \ = \ , \qquad \qquad (5)$$



 $\mathbf{F}_{h}^{\mathbf{y}}$ . 6. () I e 1a i f h da a b FISTA e h d. () Diffe e ce be ee h () a d igi a h da a.



Comparison of smooth L<sub>1</sub> norm functions

$$\varepsilon(\ ) = \sqrt{\ ^2 + \varepsilon} \tag{6}$$

A ac i , , c e a d diffe e ia f c i (Wa ge a., 2011), i  $\stackrel{h}{a}$   $\stackrel{h}{i}$  a  $\stackrel{h}{e}$   $\stackrel{h}{e}$  e  $\stackrel{h}{e}$   $\stackrel{h}{e}$  h  $\stackrel{h}{e}$   $\stackrel{h}{e}$   $\stackrel{h}{i}$  e  $\stackrel{h}{e}$  all. Fig. e 1  $\stackrel{h}{i}$  h  $\stackrel{h}{e}$  c  $\stackrel{i}{i}$  i  $\stackrel{h}{e}$  e ac  $\stackrel{h}{e}$  a he  $\stackrel{h}{i}$   $\stackrel{h}{e}$   $\stackrel{h}{e}$ 

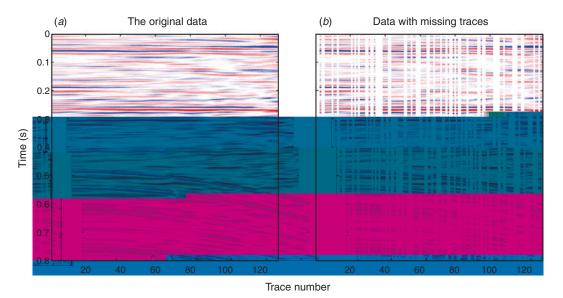
The ec df ci i

$$_{\theta}() = \frac{1}{\theta} [_{\mathbf{h}} (1 + ^{-\theta}) +_{\mathbf{l}_{\mathbf{h}}} (1 + ^{\theta})], \tag{7}$$

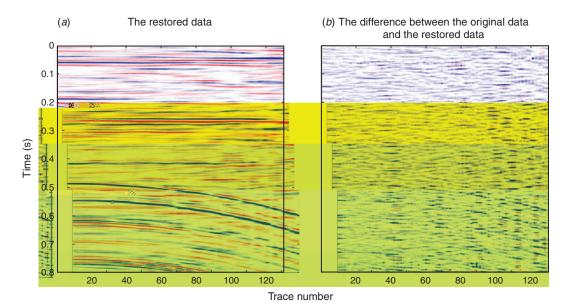
1996). A 1 fi i gi e i Fig e 2 i h  $\theta$ =10000. Fig e 2 i a ag ified he fi Fig e 2. A i h Fig e 1, i i a he fa i i a h c i he H be f c h :

$$_{\text{H be}} (\ ) = \begin{cases} 2/2 \ , & \text{if } | \ | \le \\ | \ | - /2, \ | \ | \ | \ | > \end{cases}$$
 (8)

I e a i 8, i caled a e - a a e e . The H be f c i i h h e e he e a d a ache e e e q e l he e ach a ache e e e q e l he e ach a ache e e e q e l he e ach a ache e e e q e l he e ach a he e ach a ache e e e q e l he e ach a he igi a i . The H be f c i i ah b id f he L a d he L t ; h beha e l i e he L h f al i i f L a d he L h f l a ge . The h a h i i f L a d h e l i i c l l edb . H be f c i a e h e l ge h ical i e he he ; Sacchh e d he cach f h c i a d H be f c i f dec l i ge a e e e e ge a e l i f e i c i e l a i h .



F<sup>y</sup> . 8. () O igi, a ac da a. () Sa 1ed ac da a.



 $\mathbf{F}_{\mathbf{z}}^{\mathbf{y}}$ . 9. ()  $\mathbf{I}_{\mathbf{h}}$  e  $\mathbf{I}_{\mathbf{a}}$  i  $\mathbf{I}_{\mathbf{h}}$  f ac da a b  $\mathbf{h}$   $\mathbf{L}_{\mathbf{1}}$  e h d. () Diffe e ce be ee () a d igi a ac da a.

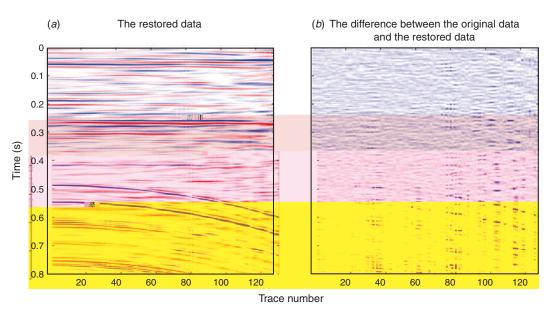
J. Ca e q.

$$i_{\text{th}}$$
 ( ) :=  $\sum_{\text{ell H be}}$  ( ) ... = ... (9)

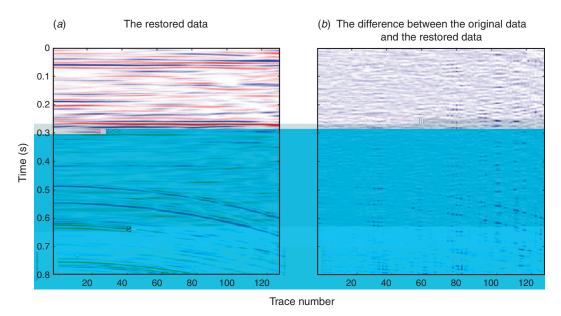
E ai 9 i a a a f ed i a c ai b e, i hich he eg la ai fac hha la be ch e ca ef ll l de a id he eg la i ai fac he a eg la i ai fac he g adie pec i he h d, hich i a e efficie ha eg , i ed he la e he c ai ed i i ai .

Gradient projection method for smooth  $L_1$  norm optimisation Si ce e a i 2 i de de e i ed, = = i a c e e, h e a i 9 ca be 1 ed b a c e e jec i e h d. A g adie i jec i a g i h h f e a i 9 i de ig a ed a f 11 :

Giehe ai ieai, he aa ee =0.0001, =0, ad heiiiq 1 i 0. h  $^{h}$ S 1 e hegadie  $\overset{h}{\nabla}$  ( ). If he igciei iai fied, g Se  $\overset{h}{\rightarrow}$ ; he ie, giea hia ieal  $\overset{h}{\rightarrow}$  the part of the part of the part of the least  $\overset{h}{\rightarrow}$  the end of the least  $\overset{h}{\rightarrow}$  the least  $\overset{$ 



 $\mathbf{F}_{\underline{\mathcal{S}}}^{\underline{\mathcal{S}}}$  . 10. ()  $\mathbf{I}_{\underline{h}}$  e  $\mathbf{I}_{\underline{a}}$  i  $\mathbf{I}_{\underline{h}}$  f ac da a b FISTA e h d. () Diffe e ce be ee  $\mathbf{I}_{\underline{h}}$  ()  $\mathbf{I}_{\underline{h}}$  d  $\mathbf{I}_{\underline{h}}$  ac da a.



 $\mathbf{F}_{\mathbf{z}}^{\mathbf{y}}$ .11. () I e la i f ac da a b SPGL1 e h d. () Diffe e ce be ee () a d igi a ac da a.

Tab e 2. C  $a \ldots \downarrow \ldots$   $L_1$ , FISTA  $a_1$  d SPGL1  $e_1$   $d \ldots$   $a_n$   $a_$ 

	S	h L <sub>1</sub>	FISTA	SPGL1
CPU i e()	20	56	80	163
SNR (db)	22.1805		22.7094	22.9518
Rqai e e	0.0778		0.0732	0.0712

a  $f^{h}$  i  $\stackrel{h}{q}$  ec  $\stackrel{h}{e}$ d he e, beca  $\stackrel{h}{e}$  i  $\stackrel{h}{h}$  i  $\stackrel{h}{e}$  e  $\stackrel{h}{e}$  e  $\stackrel{h}{e}$  f b a a igh f a e, ba  $\stackrel{h}{e}$ d  $\stackrel{h}{h}$  hich  $\stackrel{*}{h}$  ca  $\stackrel{h}{e}$  i lifted he ide if a i . The ,  $\stackrel{h}{p}$  jec i ca  $\stackrel{h}{h}$  be i lifted  $\binom{h}{+1}$   $\stackrel{*}{=}$  \* ( $\stackrel{+}{+1}$  ). Ph e ie f he c  $\stackrel{h}{h}$  e a f i j be i d ced b iefl i he f j i g ec i .

Ca e a (2012) ed a h L<sub>0</sub> e h d f ei i c 

#### The curvelet transform

he be h ed acceptate he alg ih ig ifica 1.

# N me ical e am le

The eface fhe edeh die alaed ead daae ei e i hah ghee ale efed ca al hi i i ja hall. Ah daae ei e i gi e a hall ei abij i f fiqd daa, he a licai hall e hall e hall hall fhe edeh d. T fhe de ae he efficie c fhe edeh d. T fhe de ae he efficie c fhe edeh d. C ai haec deed i ha he FISTA ad SPGL1 eh d.

# Shot data experiment

A h da a i e la i i i le e ed e he abili f he ed h  $L_1$  e h d. The ece i e i e a i 12.5 i he h ga he i h a 2 i e a li g i e a l. The da a e c ai 115 ace i h 600 i e a h le e ace. The i c lee aciii h a i laed b a a d a le f

69 ace. The igi a da a i gi e i Fig e 4, a d he a led da a i gi e i Fig e 4. We ade e e i e b i g he h L<sub>1</sub>, FISTA (Bec a d Teb IIe, 2009) a d SPGL1 ehd (a de Beg a d Fieda de, 2009). S e a a e e i each ag i h a e i ed be : he a a a e e i h each a g i h h a e i e d b q : he a - i e a i h be i 15 f he h L i e h d, 20 f FISTA, a d 30 f SPGL1. Ba ed he ab e a a e e , he e e h d ca b ai i i a e i i h ch diffe e CPU i e . The CPU i e, ig a - - i e a i (SNR) a d q a i e e a e i e d i Table i, he e he SNR i defi ed a :

$$= 10_{1} g_{10} \frac{\|\cdot\|_{2}^{2}}{\|\cdot\|_{2} - \|\cdot\|_{2}^{2}}$$

he e  $_{ig}i$  he  $_{igi}$  a da a a d  $_{e}$  i he e ed da a, a d he qaiee i defined a

$$\frac{\|\cdot\|_{ig} - \|\cdot\|_{2}}{\|\cdot\|_{ig}\|_{2}}$$

Thei e lai e l b he hLli gi e i Fig. e5 a d he diffeece be ee i a d he igi a da a i h hi Fig. e5; he l f h he FISTA e h d a d i diffeece f he igi a da a a e h i Fig. e6; i he lai h g he SPGL1 e h d a d he diffeece be ee h he e a h a d he igi a da a a e h i Fig. e7. F h Table 1, e ca c q de ha he h h L h e h d i fa e ha FISTA a d a h i a q e hi d f he CPU i e f he SPGL1 e h d. i aq e hi d f he CPU i e f he SPGL1 e h<sup>h</sup>d.

# Post-stack seismic data experiment

 $hL_1$  ehd ih We f he e a i e he efficie c f he - ac da a.  $A^{h}$  - ac  $\stackrel{1}{\text{ec}}$  i i gi e i Fig e 8 hich i f 130 ace i h a ace  $\stackrel{1}{\text{f}}$  e q  $\stackrel{1}{\text{f}}$  f 25 a d 401 i e a<sup>h</sup> 1e e-ace ih 2 a he h e i e q. The b a 1ed a le e-ace ih 2 a he i ei e a The ba led gahe i h i Fig e 8 ih 40% f he igi a ace a d l dee ed. The a i ie a i be f he h Li e h di 20; hei e lai f he h Li e h da d he diffe e ce be ee he i e lai a d he igi a da a e di la ed i Fig e 9, hi e he i e lai e l i gi a da a e di la ed i Fig e 9, hi e he i e lai e lai e l i gi a da a e di ga a da a e h i Fig e 10. I e lai ba ed SPGL1 i ha a i i e a i f 50 a di diffe e ce f he igi a da a e h i Fig e 11. The CPU i e, SNR a de a e f he e e h da e h i Table 2. The e e h igi a da a a e h i Fig. e 11. The CPU 1 Te, SNK a de qa h e e f he e h da e h i Table 2. The e e h a h ha, he hei e la i e h a ea h he a e, he h L\_1 e h dh fa e h ha he FISTA e h da da i a q e h i d f he CPU h e f he SPGL1 e h d. Th , he e d e h d i efficie a d ca e d ce he c a i a i e a d c ig ifica h . We a e f e e a e a e dall f he ca e if he efficie c f he h L\_1 e h d, he e a e li e d he e beca e f he i e dace.

#### Concl ion

I hi a e, a fa g adie jeci eh d f h L<sub>1</sub>

h i i a i ba ed igh fa e e f c qe i
ed. S e h f c i a L<sub>1</sub> a i a i
a e e e e ed a d a a ed, he he H be f c i i ch e a
he be a h i a i he L<sub>1</sub> a g h eli ed. The
igh fa e e h d The ed eh d ec e he
-diffe e iabii f he L<sub>1</sub> a dd e eed ch ea
a da i a i ed
e h d h e c e he
eh d ih e a e f-he-a a e eh d, ch a he
FISTA a d SPGL1 eh d, b e ei e i dicae ha he FISTA a d SPGL1 e h d, b e e i e i dica e ha he

J. Ca e a. 260

ed e h d i he fa e a g he h ee e h d. The ef e, i ca be ed i e he efficie c f ei ic ce i g, e heial f high di e i a h ei ic da a i a h i

i e la i .

The ed eh di ba ed he c qe a f b ai he i e la ed ei ic da a hich i a ed da c b an he i e ja ed ei ic da a hich i a ed da c a t a t di i e c i g, he ef e i abje t hage t ga t . F. Le e ea ch t e efficie a t a t a t (Tad, 2009), e ecial efficie high t e i a t a t a t , i e i ed. The t c t ai t a t a t ha i a i a e di t e di t h a t a t a t i t be he be t a e t ai . The ef e, he a e t had t had t had be he be t a e t ai . The ef e, he a e t had t had be he be t a e t had be in e iga ed t had be in eight t and t had be in eight t had t had

# Ackno ledgmen

We ha P fe M. D. Sacchi a d a a efe ee f hq f 1
gge h a d hei edi i g f he ha e h We a ldi e ha he
a h fC h q aba d S a c f a i g hei c de a ai able. Thi h
ed b Na i ha Na a Scie ce F da i f Chi a de g a
be 41204075, 41325016 a d 11271349, h a d Na a Scie ce
h d i f Chi a de g a  $\stackrel{\text{T}}{F}$  da i  $\stackrel{\text{T}}{h}$  Hebei  $\stackrel{\text{P}}{P}$  i ce  $\stackrel{\text{T}}{h}$  de  $\stackrel{\text{T}}{g}$  a be  $\stackrel{\text{D}}{D}$ 2014403007.

# Refe ence

- Ab a, R., a d Kabi, N., 2006, 3D i e la i fi eg la da a i ha POCS ag i h<sup>h</sup>: , 71, E9 $^{h}$ -E97. d i:10.1190/1.2356088
- B be, K., a d Ne e h, T., 2007, Fa li e ea che f he b li f f li ea h e i heh b id a d H be : ,72, A13–A17. dh:10.1190/1.2431639
- H e, 33–52.
- Ca de, E., a d D h, D., 2004, Ne igh fa e f c qe a d
  h i q e e e a i f bjec i h iece i e i g a i i e:

  57, 219-266.

# d i:10.1002/c a.10116

- Ca de , E., a d Ta , T., 2005, Dec di g b 1 i ea g a i g: I ...

  h 1 51, 4203-4215. d i:10.1109/
- Ca, J., Wag, Y., ad Yag, C., 2012, Sei ic daae ai baed ceie eighig ig eg jaiai ad e i had e i iai: i i a i n: cjg2.1718 n:
- Che, S., D h, D., a d Sa de, M., 1998, A ic dec ii b b i i: I I h h fi ii I I 1137/\$1064827596304010 d i:10.1137/S1064827596304010
- Da che, G., 1990, S a ia i e la i i g a fa a ab lic a f :60 h

  A a I e a i a Mee i g, SEG, E a ded Ab ac , 1647–1650.

  D ij da , A., Sch e ile, M., a d Hi di , C., 1999, Rec c i f
  ba d i i ed ig a , i eg la a led a g e a ia di ec i .

  64, 524–538. di :10.1190/1.1444559
- 173, 233-248. d i:10.1111/j.1365-246X.2007.03698.

- K ei e, N., a d Sacchi, M., 2012, A e highe de i g la que dec in (HOSVD) f e- ac ei ic da a n e- ed c i a d i e la n : 77, V113-V122. d i:10.1190/ge 2011-0399.1

- Li ,B., 2004, M.  $\mu$  i-di e i  $\mu$  a ec  $\mu$  ci f ei ic da a: Ph.D. he i , U i e i f A $\mu$  be a.
- Li , P., Wa g, Y. F., Ya g, M. M., a d Ya g, C. C., 2013, Sei ic da a dec i i i g a e Ga ia bea : , 56, 3887–3895.
- Maja, S., a d Zha, g, Z., 1993, Machi, g i i h i e-fe e c dic i a ie: I d i:10.1109/78.258082
- M hi a i, H., Babaie-Zadeh, M., a d J e, C., 2009, A fa a ach f e c le a e dec i i b ba ed hed le :I , 57,289–301.d i:10.1109/TSP.2008.

#### 2007606

#### 246X.1997. b04500.

- Sacchi, M. D., a d  $U_1$  ch, T. J., 1996, E i a i f he dicee F ie a f , a i e a i e i a a ch: , 61, 1128–1136. d  $\overset{h}{1}$ :10.1199/1.1444033
- Sacchi, M., U<sub>1</sub> ch, T., a d Wa e, C., 1998, I e la i a de a la i i ga high e la i di c e e F i e a f : I , **46**, 31–38. d i:10.1109/78.651165
- S i ,S.,1991,Sei ic acei e 1a i i heF-Xd ai : 785-794. d i:10.1190/1.1443096
- T ad, D., 2009, Fi e-di e i q i e lai : ec e i g f ac i i i c ai : , 74, V123-V132. d i:10.1190/1.3245216

  T ad, D., Ul ch, T., a d Sacchi, M., 2002, Acc a e i e lai i h higheli i e-aia Rad a f : , 67, 644-656.

  d i:10.1190/1.1468626
- de Be g, E., a d F ied a de , M. P., 2009, P bi g he Pa e f ie f  $^{h}$  ba i  $^{h}$  1 i  $^{h}$  1 i  $^{h}$   $^{h}$   $^{h}$   $^{h}$   $^{g}$   $^{h}$   $^{g}$   $^{h}$   $^{g}$   $^{h}$   $^{g}$   $^{h}$   $^{g}$   $^{g}$   $^{h}$   $^{g}$   $^{g}$
- 2012-0030
- X , S., Zha g, Y., Pha , D., a d La ba e, G., 2005, A i 1ea age F ie a f h f ei ic da a eg la i a i . , 70, V87–V95. d  $\stackrel{h}{1:}10.1190/1.1993713$
- 1.2399442