



a priori

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Abstract

fi \mathbf{H} , a priori l^1 , l^2 , 318, J, fi

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1. Introduction

M, 1976; ..., 1977). The density profile, $n(r)$, is given by the following formula (..., 1983; ..., 1974;

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E-mail address: ..., ..., @ ..., ... (..., ...).

flame (Hanson & Dickey, 1929), $\int_0^\infty r^2 n(r) dr = \tau_\infty$, where $\tau_\infty = \beta \lambda^{-\alpha}$, $\tau_\infty = \tau(\lambda, \eta)$, $\beta = \beta(\lambda, \eta)$, $\alpha = \alpha(\lambda, \eta)$. The function $n(r)$ is the particle size distribution function, $\tau(\lambda, \eta)$ is the total extinction coefficient, β is the extinction coefficient per unit area, α is the extinction exponent, λ is the wavelength, and η is the extinction parameter.

$$\tau_\infty(\lambda) = \int_0^\infty \pi r^2 Q_\infty(r, \lambda, \eta) n(r) dr + \varrho(\lambda), \quad (1)$$

The extinction coefficient per unit area β is related to the extinction coefficient τ by $\beta = \tau/\pi r^2$. The extinction parameter η is related to the extinction coefficient τ by $\eta = \ln(\tau/\tau_\infty)/\ln(\lambda/\lambda_\infty)$. The extinction coefficient per unit area β is related to the extinction coefficient τ by $\beta = \tau/\pi r^2$. The extinction parameter η is related to the extinction coefficient τ by $\eta = \ln(\tau/\tau_\infty)/\ln(\lambda/\lambda_\infty)$.

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$$n(r) = \dots \quad (1).$$

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3. Problem formulation

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3.1. Operator equations of the first kind

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$$\int_a^b k(x, y)n(y) \cdot y = o(x), \quad (2)$$

[a, b] fl

fl

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fl

$$\int_a^b k(x, y)n(y) \cdot y + \varrho(x) = o(x) + \varrho(x) = d(x), \quad (3)$$

fl

fl

fl

fl

fl

$$(Kn)(\lambda) := \int_0^\infty k(r, \lambda, \eta)n(r) \cdot r; k(r, \lambda, \eta) = \pi r^2 Q(r, \lambda, \eta); F \subset O \subset H, \quad (4)$$

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4. Theoretical development

4.1. Solving for an efficient a priori information in l^1 space

$$\text{fi} \quad \text{,} \quad \text{.} \quad (4)$$

$$\text{fi} \quad \text{.}$$

$$(2003)$$

$$\mathbf{H} \quad \text{,}$$

$$l^1$$

$$\|\vec{n}\|_{l^1} \quad \quad \quad \mathcal{K}\vec{n} = \vec{d}, \quad \vec{n} \geq 0. \quad (7)$$

$$(7)$$

$$e \quad \vec{n} \quad \quad \quad \mathcal{K}\vec{n} = \vec{d}, \quad \vec{n} \geq 0, \quad (8)$$

$$e \quad \vec{n} \quad \quad \quad \frac{1}{\|\vec{n}\|_{l^1}} \quad \quad \quad \text{fi} \quad \text{.} \quad (8)$$

$$S = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} \geq 0\}.$$

$$S; \quad \quad \quad \text{.} \quad (8)$$

$$(8)$$

$$\vec{d} \quad \vec{z} \quad \quad \quad s = e - \mathcal{K} \quad \vec{z} \geq 0. \quad (9)$$

$$(\vec{n}, \vec{z}, s)$$

$$\mathcal{K}\vec{n} = \vec{d}, \quad (10)$$

$$\mathcal{K} \quad \vec{z} + s = e, \quad (11)$$

$$\tilde{S} \tilde{F} e = 0, \quad (12)$$

$$\vec{n} \geq 0, \quad s \geq 0, \quad (13)$$

$$\tilde{S} = \text{.} \quad , (s_1, s_2, \dots, s_N), \quad \tilde{F} = \text{.} \quad , (n_1, n_2, \dots, n_N).$$

$$\text{fi} \quad \text{,} \quad \text{.} \quad (10)$$

$$\|\vec{d} - \mathcal{K}\vec{n}_k\| \leq \|\mathcal{K} \quad \vec{z}_k + s_k - e\| \quad \text{,} \quad \vec{n}_k > 0 \quad \text{,} \quad s_k > 0. \quad k$$

$$\text{fi} \quad \text{,} \quad \text{.} \quad (11)$$

$$[\vec{n}_k, \vec{z}_k, s_k] \quad \text{,} \quad \beta_k \in [0, 1], \quad (12)$$

$$\begin{bmatrix} \mathcal{A}_{\vec{n}} & \mathcal{A}_{\vec{z}} & \mathcal{A}_s \end{bmatrix} \begin{bmatrix} \vec{n} \\ \vec{z} \\ s \end{bmatrix} = \begin{bmatrix} \vec{d} - \mathcal{K}\vec{n}_k \\ e - \mathcal{K}\vec{z}_k - s_k \\ \beta_k \mu_k e - \tilde{S}_k \tilde{F}_k e \end{bmatrix}, \quad (14)$$

$$\beta_k = (1/N)\vec{n}_k s_k, \quad \tau_k = \frac{\vec{n}_k}{\beta_k}, \quad \vec{n}_{k+1} := \vec{n}_k + \tau_k \mathcal{A}_{\vec{n}}, \quad \vec{z}_{k+1} := \vec{z}_k + \tau_k \mathcal{A}_{\vec{z}}, \quad s_{k+1} := s_k + \tau_k \mathcal{A}_s. \quad (15)$$

4.2. Damped Gauss–Newton method

$$R(\vec{n}) = \mathcal{K}\vec{n} - \vec{d}, \quad J[\vec{n}] = \|R(\vec{n})\|_2^2, \quad (16)$$

$$g(\vec{n}) = \mathcal{K}(\mathcal{K}\vec{n} - \vec{d}), \quad H = \mathcal{K}^\top \mathcal{K}. \quad (17)$$

$$s_k = \vec{n}_{k+1} - \vec{n}_k, \quad s_k = \frac{1}{2} \|R(\vec{n}_k) + R'(\vec{n}_k)s\|^2, \quad (18)$$

$$s_k = -R'(\vec{n}_k)^{-1} g_k, \quad (19)$$

$$\vec{n}_{k+1} = \vec{n}_k + s_k, \quad (20)$$

$$g_k = g(\vec{n}_k).$$

$$s_k = -\gamma_k H^{-1} g_k, \quad (21)$$

$$\gamma_k = \arg \min_{\gamma} \phi(\gamma) := J[\vec{n}_k + \gamma s_k]. \quad (22)$$

4.3. Regularization by incorporating an efficient a priori information

$$s_k = -\gamma_k (H + \alpha_k L)^{-1} g_k, \quad (23)$$

$$L = \frac{1}{2} \|s_k\|_2^2 + \alpha_k \|s_k\|_1, \quad \alpha_k \text{ is a regularization parameter.} \quad (H \text{ is the operator, } s_k \text{ is the residual vector, } \alpha_k \text{ is the regularization parameter, and } L \text{ is the regularization term.)}$$

$$s_k = -\gamma_k (H + \alpha_k L)^{-1} (g_k + \alpha_k (\vec{n}_k - \vec{n}_0)), \quad (24)$$

\vec{n}_0 is the initial value of \vec{n} , which is obtained from the a priori information. The α_k is determined by the method proposed by Hansen et al. (1994).

4.4. Aerosol particle size distribution function retrieval

(1978), ..., $h(r) \dots f(f) : n(r) = h(r)f(r)$, ..., $h(r) \dots r$, ..., $f(r) \dots$

$$\tau_-(\lambda) = \int_a^b [k(r, \lambda, \eta) h(r)] f(r) \, r, \quad (25)$$

$$k(r, \lambda, \eta) = \pi r^2 Q(r, \lambda, \eta) - k(r, \lambda, \eta) h(r) \quad (\Xi f)(r) = \tau(\lambda). \quad (26)$$

Ξ \mathcal{K} .

$$\alpha \rightarrow \alpha' \text{ and } \beta \rightarrow \beta' \text{ are such that } \alpha' \beta' = \alpha \beta \text{ and } \alpha' \beta' \in \mathcal{A}.$$

¹ See also the discussion in *Mathematical Methods in the Social Sciences*, ed. C. C. Holt, R. A. Pollak, and H. H. Raiffa (New York: John Wiley & Sons, 1957), pp. 1-10.

(2006). The effect of the α parameter on the performance of the model.

$$\alpha_k = \alpha_0 \cdot \zeta^{k-1}, \quad (27)$$

¹⁰ See also the discussion in Section 3.2 of the main text, where we show that $\alpha_0 = 0.1$; $\xi \in (0, 1)$ and $\beta \in (0, 1)$ are consistent with the observed data.

¹ The term α_k is the k th element of the vector α .

¹ See also the discussion of the relationship between the two in the section on the "Ergonomics of Workstation Design".

$$\begin{array}{cccccc} 1 + \frac{1}{h_r^2} & -\frac{1}{h_r^2} & 0 & \cdots & 0 \\ -\frac{1}{h_r^2} & 1 + \frac{2}{h_r^2} & -\frac{1}{h_r^2} & \cdots & 0 \end{array}$$

$$L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (28)$$

$$0 \quad \cdots \quad -\frac{1}{h_r^2} \quad 1 + \frac{2}{h_r^2} \quad -\frac{1}{h_r^2}$$

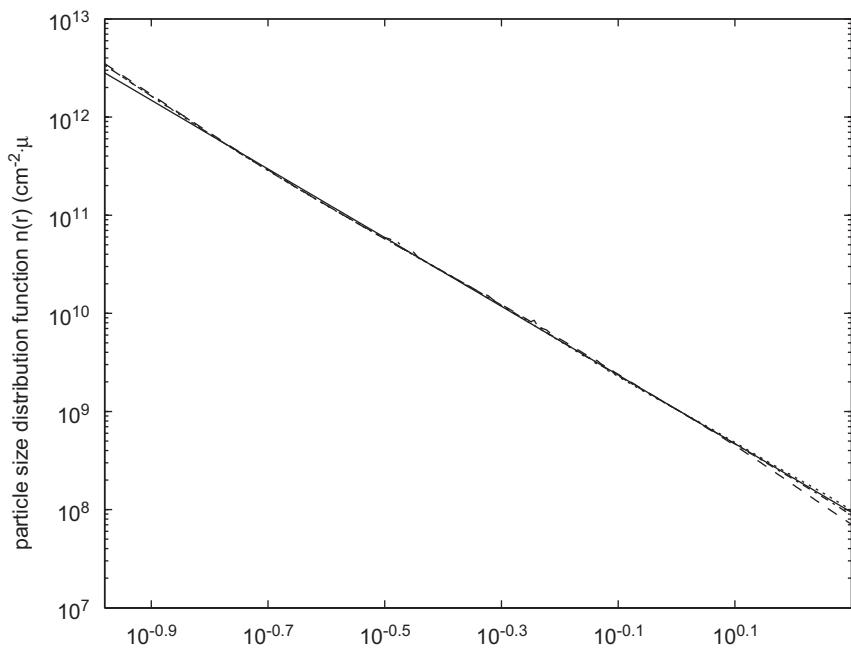
$$\begin{bmatrix} 0 & \cdots & 0 & -\frac{1}{h_r^2} & 1 + \frac{1}{h_r^2} \end{bmatrix}$$

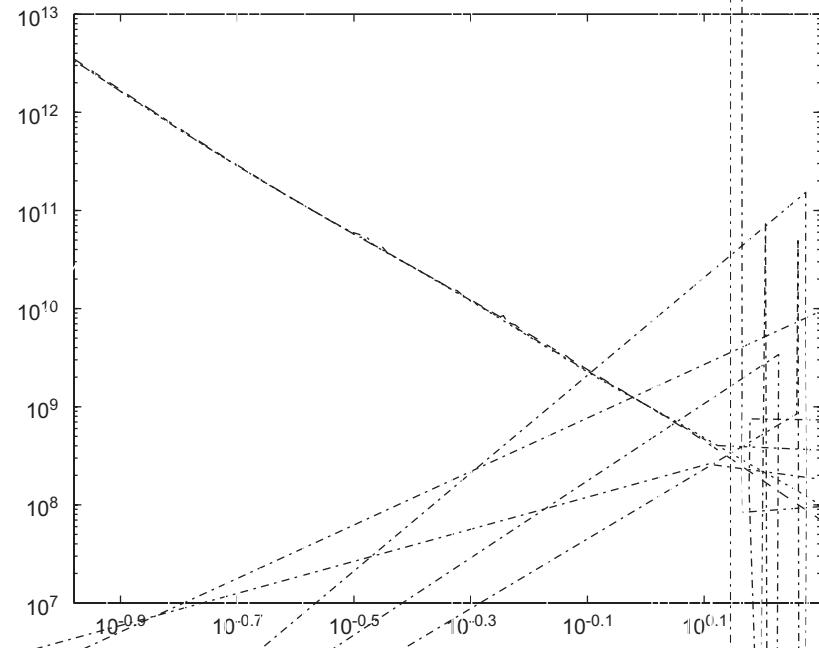
¹⁰ For a discussion of the relationship between the two, see D. S. G. Pollock, 'The Economics of the Slave Trade' (1996).

5. Numerical experiments

5.1. Theoretical simulation

³ See, e.g., *U.S. v. Ladd*, 10 F.2d 100, 103 (1st Cir. 1925) (holding that a conviction for mail fraud was not barred by the statute of limitations); *U.S. v. Gandy*, 10 F.2d 103, 105 (1st Cir. 1925) (same).



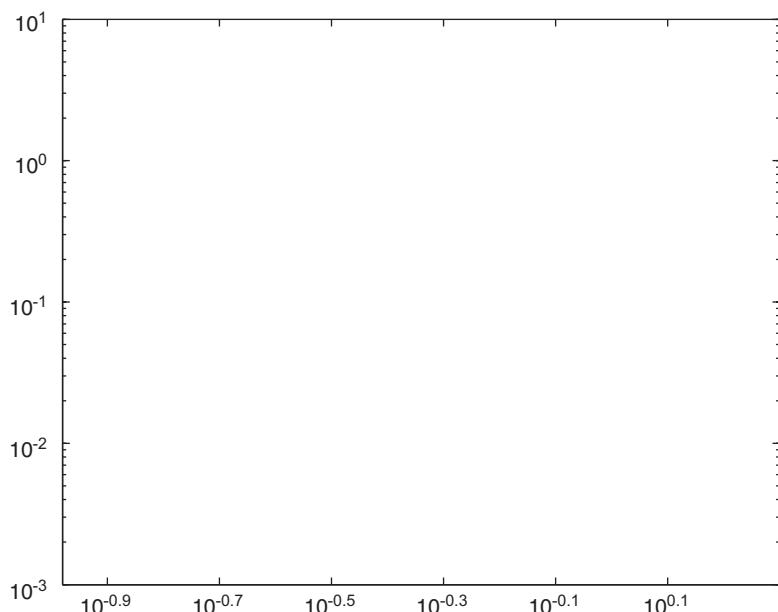


1
 Fig. 1. The effect of η on the a priori solution.

	$\eta = 1.45 - 0.00i$	$\eta = 1.45 - 0.03i$	$\eta = 1.50 - 0.02i$
$\delta = 0.005$	1.6443×10^{-4}	1.2587×10^{-4}	2.2773×10^{-4}
$\delta = 0.01$	1.6493×10^{-4}	1.2720×10^{-4}	2.2847×10^{-4}
$\delta = 0.05$	1.6996×10^{-4}	1.3938×10^{-4}	2.3504×10^{-4}

2
 Fig. 2. The effect of η on the a priori solution.

	$\eta = 1.45 - 0.00i$	$\eta = 1.45 - 0.03i$	$\eta = 1.50 - 0.02i$
$\delta = 0.005$	17	13	16
$\delta = 0.01$	17	13	16
$\delta = 0.05$	17	13	16



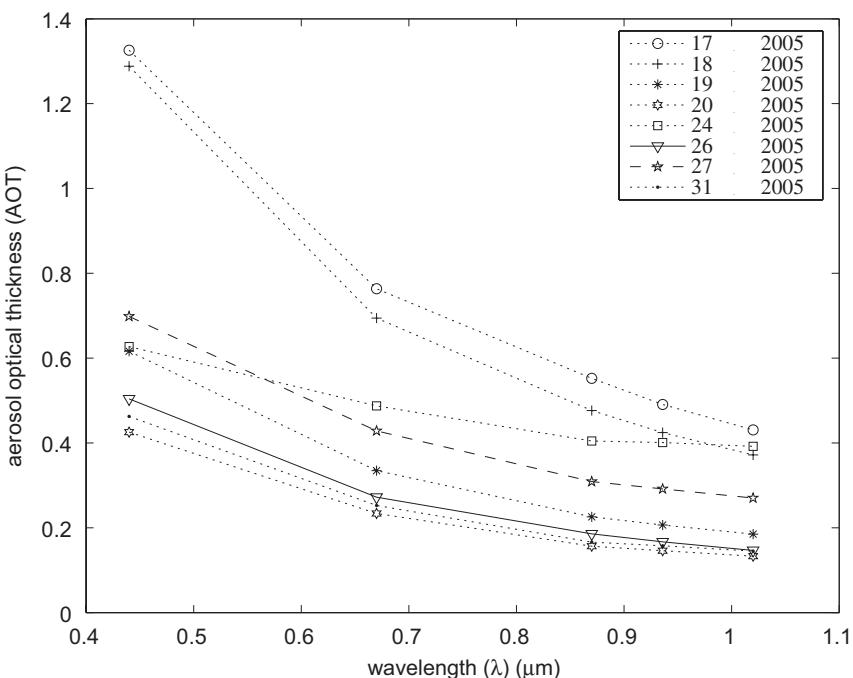


Fig. 6. Comparison of aerosol optical thickness (AOT) between site 17 and site 31 (\overline{M}), 2005.

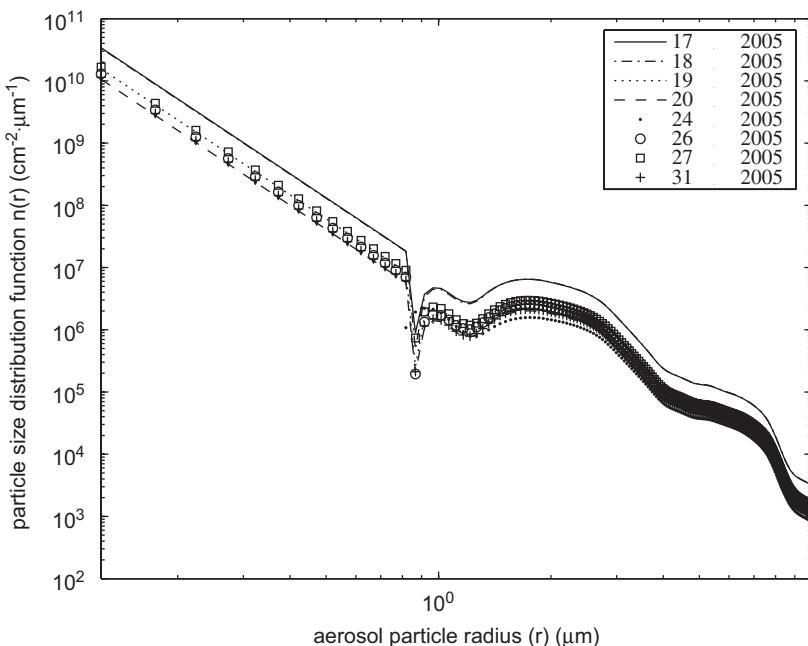
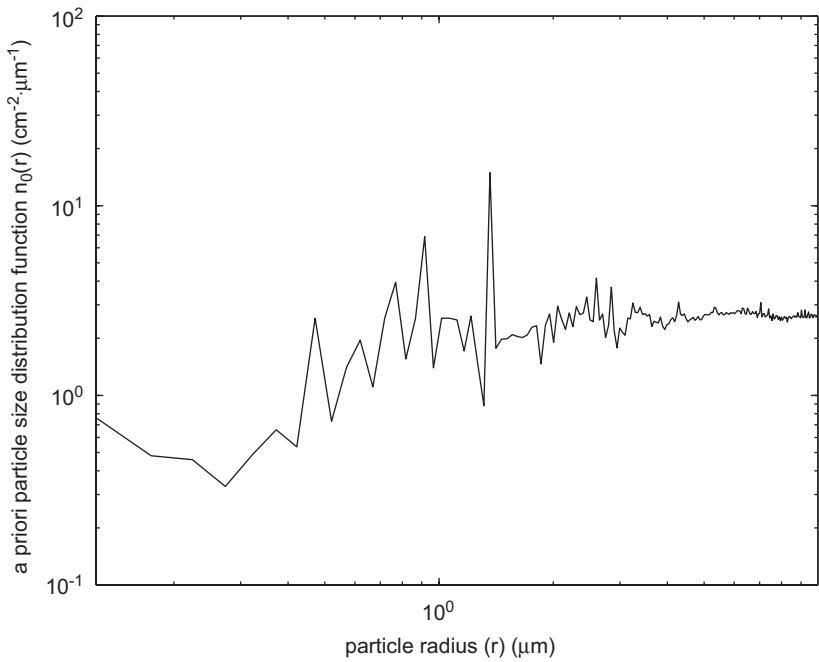


Fig. 7. Comparison of particle size distribution function ($n(r)$) between site 17 and site 31 (\overline{M}), 2005.



¹⁸ *A priori* n_0 (M), 2005.

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Appendix A. Conception of regularization

3.1.1. *Conception of regularization*. Let K be a linear operator from H to F , F is a Hilbert space, H is a Banach space, O is a closed linear subspace of H , $\text{fi}(K)$ is the range of K , $\text{fi}(K^*)$ is the range of K^* , $\text{fi}(K^*)^\perp$ is the orthogonal complement of $\text{fi}(K^*)$ in F , $\text{fi}(K^*)^\perp \cap \text{fi}(K) = \{0\}$.

$$Kn = o, \quad (A.1)$$

$$K^*n \in \text{fi}(K^*)^\perp \cap \text{fi}(K^*) = \{0\}, \quad H_n \in F, \quad F \subset O.$$

$$\Pi^\alpha : O \rightarrow F, \quad \alpha > 0, \quad (A.2)$$

$$\Pi^\alpha K n = n, \quad \forall n \in F, \quad (A.3)$$

$$\Pi^\alpha K^* = K^* \Pi^\alpha, \quad \Pi^\alpha = \text{fi}(K^*)^\perp \cap \text{fi}(K^*)^\perp.$$

$$\Pi^\alpha = (K^* K + \alpha L)^{-1} K^*, \quad (A.4)$$

$$L = K^* K + \alpha I, \quad K^* \in \text{fi}(K^*)^\perp, \quad K, \alpha \in (0, 1).$$

Appendix B. Conception of *a priori* information

$$\frac{1}{2} \|Kn - o\|^2 + \frac{1}{2}\alpha \|L^{1/2}n\|^2 \quad (B.1)$$

$$n^\alpha = \Pi^\alpha o. \quad (B.2)$$

$$\{ \frac{1}{2} \|Kn - o\|^2 + \alpha \Omega(n - n_0) : n \in F \}, \quad (B.3)$$

$$\begin{aligned} n_0 &\in \arg \min_{n \in F} \{ \frac{1}{2} \|Kn - o\|^2 + \alpha \Omega(n - n_0) \}, \\ \text{a priori } n_0 &\in \arg \min_{n \in F} \{ \frac{1}{2} \|Kn - o\|^2 + \alpha \Omega(n - n_0) \}, \end{aligned}$$

$$\text{a priori } n_0 \in \arg \min_{n \in F} \{ \frac{1}{2} \|Kn - o\|^2 + \alpha \Omega(n - n_0) \}. \quad (B.1)$$

a priori : (1) $n_0 \in S_p = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} > 0\}$; (2) $\vec{n} \in S_D = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} > 0, D\vec{n} = \mu e\}$.

Appendix C. Computing an *a priori* by searching for an interior point solution

a priori : (1) $n_0 \in S_p = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} > 0\}$; (2) $\vec{n} \in S_D = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} > 0, D\vec{n} = \mu e\}$.

$$c \cdot \vec{n} - \mu \sum_{j=1}^N n_j = \mathcal{K}\vec{n} - \vec{d}, \quad \vec{n} \geq 0, \quad (C.1)$$

$$\vec{d} \cdot \vec{z} - \mu \sum_{j=1}^N z_j = \mathcal{K}\vec{z} + s = c, \quad s \geq 0. \quad (C.2)$$

$$c \cdot \vec{n} - \mu \sum_{j=1}^N n_j = \mathcal{K}\vec{n} - \vec{d}, \quad \vec{n} \geq 0. \quad (C.3)$$

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$$\lim_{n_j \rightarrow 0} (-\mu \sum_{j=1}^N n_j) = \infty. \quad (C.4)$$

$$L(\vec{n}, \vec{z}) = c \cdot \vec{n} - \mu \sum_{j=1}^N n_j - \vec{z} \cdot (\mathcal{K}\vec{n} - \vec{d}). \quad (C.5)$$

$$\frac{\partial L}{\partial n_j} = c_j - \mu n_j^{-1} - \mathcal{K}_{:j} \vec{z}, \quad \frac{\partial L}{\partial z_i} = \vec{d}_i - \mathcal{K}_{i:\vec{n}}, \quad (C.6)$$

$$\mathcal{K}_{:j} \quad j = 1, \dots, N, \quad \mathcal{K}_{i:} \quad i = 1, \dots, N, \quad \mathcal{K}.$$

$$L(\vec{n}, \vec{z}) = c - \mu D^{-1} e - \mathcal{K} \cdot \vec{z}, \quad L(\vec{n}, \vec{z}) = \vec{d} - \mathcal{K} \cdot \vec{n}, \quad (C.7)$$

$$D = \begin{bmatrix} n_1 & 0 & \cdots & 0 \\ 0 & n_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & n_N \end{bmatrix}. \quad (C.8)$$

$$\mathcal{K} \cdot \vec{z} = c - \mu D^{-1} e, \quad \mathcal{K} \cdot \vec{n} = \vec{d}, \quad \vec{n} > 0. \quad (C.9)$$

$$\text{fi } s = \mu D^{-1} e, \quad \mathcal{K} \cdot \vec{z} + s = c, \quad \mathcal{K} \cdot \vec{n} = \vec{d}, \quad Ds = \mu e, \quad \vec{n} > 0. \quad (C.10)$$

$(\vec{n}, \vec{z}, s) \in S_D$, $\vec{n} \in S_p$, $\vec{z} \in S_D$, $s \geq 0$, $\vec{n} \in S_p = \{\vec{n} : \mathcal{K}\vec{n} = \vec{d}, \vec{n} > 0\}$, $\vec{z} \in S_D = \{\vec{z} : \mathcal{K}\vec{z} + s = c, s = \mu D^{-1} e > 0\}$.

$$c \vec{n} - \vec{d} \vec{z} = \vec{n} s = \mu N, \\ F(\vec{n}, \vec{z}, s) = \begin{bmatrix} \mathcal{K} \vec{n} - \vec{d} \\ \mathcal{K} \vec{z} + s - c \\ Ds - \beta \mu e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (11)$$

$\beta \in [0, 1]$.

$$(\vec{n}_k, \vec{z}_k, s_k) \text{ is a solution, } F(\vec{n}, \vec{z}, s) = 0$$

$$F(\vec{n}_k, \vec{z}_k, s_k) \begin{bmatrix} d_{\vec{n}} \\ d_{\vec{z}} \\ d_s \end{bmatrix} = -F(\vec{n}_k, \vec{z}_k, s_k). \quad (12)$$

$$F(\vec{n}_k, \vec{z}_k, s_k) = \begin{bmatrix} \mathcal{K} & 0 & 0 \\ 0 & \mathcal{K} & I \\ S_k & 0 & D_k \end{bmatrix}, \quad (13)$$

$$\begin{bmatrix} \mathcal{K} & 0 & 0 \\ 0 & \mathcal{K} & I \\ S_k & 0 & D_k \end{bmatrix} \begin{bmatrix} d_{\vec{n}} \\ d_{\vec{z}} \\ d_s \end{bmatrix} = \begin{bmatrix} \vec{d} - \mathcal{K} \vec{n}_k \\ c - \mathcal{K} \vec{z}_k - s_k \\ -D_k s_k + \beta \mu_k e \end{bmatrix}. \quad (14)$$

Algorithm (Computing an interior point solution).

- (1) Initial values: $(\vec{n}_1, \vec{z}_1, s_1)$, $\vec{n}_1, \vec{z}_1 > 0$, $\varepsilon_{\vec{n}}, \varepsilon_{\vec{z}}, \varepsilon_s > 0$;
- (2) $k = 1, \dots, \infty$;
- (3) $r_{\vec{n}}^k = \vec{d} - \mathcal{K} \vec{n}_k$,

$$r_{\vec{z}}^k = c - \mathcal{K} \vec{z}_k - s_k,$$

$$\mu_k = \frac{1}{N} \vec{n}_k s_k;$$

$$(4) \|r_{\vec{n}}^k\| \leq \varepsilon_{\vec{n}}, \|r_{\vec{z}}^k\| \leq \varepsilon_{\vec{z}}, \|\mu_k\| \leq \varepsilon_s, \dots;$$

$$(5) \beta \in (0, 1) \quad (14);$$

$$(6) \dots$$

$$\tau = \tau \geq 0 \left\{ \begin{bmatrix} \vec{n}_k \\ s_k \end{bmatrix} + \tau \begin{bmatrix} d_{\vec{n}} \\ d_s \end{bmatrix} \geq 0 \right\}; \quad (15)$$

$$(7) \theta \in (0, 1),$$

$$\tau := \{\theta \tau, 1\};$$

$$(8) \dots$$

$$\vec{n}_{k+1} := \vec{n}_k + \tau d_{\vec{n}},$$

$$\vec{z}_{k+1} := \vec{z}_k + \tau d_{\vec{z}},$$

$$s_{k+1} := s_k + \tau d_s.$$

Example (Computing an interior point solution).

$$-1n_1 - 2n_2 \quad n_1 + n_2 = 1, \quad n_1, n_2 \geq 0. \quad (1.16)$$

$$\vec{n} = [n_1 \ n_2], \quad c = [-1 \ -2], \quad \mathcal{K} = [1 \ 1], \quad \vec{d} = 1, \quad F(\vec{n}, \vec{z}, s) = \begin{bmatrix} \mathcal{K}\vec{n} - \vec{d} \\ \mathcal{K}\vec{z} + s - c \\ Ds - \beta\mu e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (1.17)$$

$$\vec{z} = [z_1 \ z_2], \quad s = [s_1 \ s_2], \quad D = \text{diag}(n_1, n_2), \quad \mu = \frac{1}{2}\vec{n}^T s, \quad e = [1 \ 1], \quad \beta = 0.1, \quad \vec{n} \in [0, 1].$$

$$F(\vec{n}_k, \vec{z}_k, s_k) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ s_1 & 0 & 0 & \vec{n}_1 & 0 \\ 0 & s_2 & 0 & 0 & \vec{n}_2 \end{bmatrix}.$$

$$\vec{n}^* = [0.0000 \ 1.0000], \quad s^* = 1.0042e-007, \quad c^* = -2.0000.$$

Appendix D. Damped Gauss–Newton method with line search

$$J[\vec{n}] := \frac{1}{2}\|R(\vec{n})\|^2. \quad (1)$$

$$s_k = \vec{n}_{k+1} - \vec{n}_k, \quad \vec{n}_{k+1} = \vec{n}_k + s_k.$$

$$\frac{1}{2}\|R(\vec{n}_k) + R'(\vec{n}_k)s\|^2, \quad (2)$$

$$s_k = -H^{-1}g_k, \quad \vec{n}_{k+1} = \vec{n}_k + s_k.$$

(Biegler & Grossmann, 1999), find the step size γ_k such that $J(\vec{n}_k + \gamma_k s_k) \leq J(\vec{n}_k) + c_1 \gamma_k g_k^T s_k$,

$$s_k = -\gamma_k H^{-1}g_k. \quad (3)$$

Find the step size γ_k , such that $J(\vec{n}_k + \gamma_k s_k) \leq J(\vec{n}_k) + c_1 \gamma_k g_k^T s_k$. If $J(\vec{n}_k + \gamma_k s_k) < J(\vec{n}_k)$, then $\vec{n}_{k+1} = \vec{n}_k + \gamma_k s_k$; otherwise, $\vec{n}_{k+1} = \vec{n}_k$.

$$J(\vec{n}_k + \gamma_k s_k) \leq J(\vec{n}_k) + c_1 \gamma_k g_k^T s_k, \quad (4)$$

$$c_1 \in (0, 1).$$

Find the step size γ_k such that $J(\vec{n}_k + \gamma_k s_k) \leq J(\vec{n}_k) + c_2 g_k^T s_k$. If $J(\vec{n}_k + \gamma_k s_k) < J(\vec{n}_k)$, then $\vec{n}_{k+1} = \vec{n}_k + \gamma_k s_k$; otherwise, $\vec{n}_{k+1} = \vec{n}_k$.

$$g(\vec{n}_k + \gamma_k s_k) \cdot s_k \geq c_2 g_k^T s_k, \quad (5)$$

$$c_2 \in (c_1, 1), \quad c_1 \in (0, 1). \quad (4).$$

$$(4) \quad (5) \quad \text{for } c_1 = 0.1 \quad c_2 = 0.4.$$

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