Available online at www.sciencedirect.com

ScienceDirect

ELSEVIER

38 (2007) 885 901

Journal of Aerosol Science

a priori

*,

Institute of Geology and Geophysics, Chinese Academy of Sciences, P.O. Box 9825, Beijing 100029, PR China

State Key Laboratory of Remote Sensing Science, Jointly Sponsored by the Institute of Remote Sensing Applications of Chinese Academy of Sciences and Beijing Normal University, Beijing 100101, PR China

Department of Mathematics, Liaocheng University, Shandong 252059, PR China

School of Geography, Beijing Normal University, Beijing 100875, PR China

12 2006; 6 2007; 12 2007

Abstract

fi . , , . H , *a priori* , , , / fi . , . fi l^1 *a priori* , l^2 . 318 . fi .

PACS: 02.30. ; 42.68. ; 42.68. ; 92.60. ; 84.40. ; 02.60.

Keywords: ; ; ; *A priori*; l^1 l^2 ;

1. Introduction

, (& H , 1983; , 1974; M , 1976; , 1977). , $n(r)$, . , . fi

* 9825, 100029, .
.: +86 10 6200 8008; : +86 10 6201 0846.
E-mail address: @ ( ).

0021-8502/$ - © 2007 .
:10.1016/ .2007.06.005

fi (1929), ( ) $\tau = \beta\lambda^{-\alpha}$, τ , β fi , α fl .

fi

$$\tau(\lambda) = \int_0^\infty \pi r^2 Q(r, \lambda, \eta) n(r) r + \varrho(\lambda), \quad (1)$$

r ; $n(r)$ (.,); η ; λ ; $\varrho(\lambda)$ / Q (r, λ, η) fi M . fl , . , .

$n(r)$, , ., (, 1962; , 1963, 1975), (, 2001), (, 1970; , , & , 1995; & , 1994; , 1975; & , 1969), (

318 , ,
fi $n(r)$. (1). ,
fi .

3. Problem formulation

fl
fi .

3.1. Operator equations of the first kind

fi (&
, 1989; & , 1996; , 1975; & , 2000; , 2007)

$$\int_a^b k(x,y)n(y)\,y=o(x), \tag{2}$$

$[a,b]$, $o(x)$
- , $x=$ r $k(x,y)$ (,)
fi , .
. H , (2) $o(x)$
$n(x)$

$$\int_a^b k(x,y)n(y)\,y+\varrho(x)=o(x)+\varrho(x)=d(x), \tag{3}$$

$\varrho(x)$. ,
fi $n(x)$.
. (1)

$$K: F \longrightarrow O,$$
$$(Kn)(\lambda)+\varrho(\lambda)=d(\lambda), \tag{4}$$

$(Kn)(\lambda):=\int_0^\infty k(r,\lambda,\eta)n(r)\,r$; $k(r,\lambda,\eta)=\pi r^2 Q\ (r,\lambda,\eta)$; F
; O . F O H .
τ (λ) . (1) , / . H , $d(\lambda)$ -
. , . (4)

$$Kn+\varrho=d. \tag{5}$$

3.2. Discrete formulation in finite spaces

. (4) fi fi ,
$n(r)$.
(., 2006), fi fi $\{r_j\}_{j=1}^N$
$[a,b]$.
$\mathscr{K}=(\mathscr{K}_{ij})_{M\times N}$, $\vec{n}$, $\vec{\varrho}$ $\vec{d}$,

$$\mathscr{K}\vec{n}+\vec{\varrho}=\vec{d}. \tag{6}$$

.

4. Theoretical development

4.1. Solving for an efficient a priori information in l^1 space

fi , . (4)

fi .

, , fi .

, , (2003) ,

. H ,

,

.

;

.

, ,

l^1

$$\|\vec{n}\|_{l^1} \quad \mathscr{K}\vec{n} = \vec{d}, \quad \vec{n} \geqslant 0. \tag{7}$$

. (7)

$$e \quad \vec{n} \quad \mathscr{K}\vec{n} = \vec{d}, \quad \vec{n} \geqslant 0, \tag{8}$$

e 1. (8) (7).

. (8) fi .

l^1

$$S = \{\vec{n} : \mathscr{K}\vec{n} = \vec{d}, \vec{n} \geqslant 0\}.$$

S; (, 1997).

. (8)

$$\vec{d} \quad \vec{z} \quad s = e - \mathscr{K} \quad \vec{z} \geqslant 0. \tag{9}$$

, $(\vec{n}, \vec{z}, s)$

$$\mathscr{K}\vec{n} = \vec{d}, \tag{10}$$

$$\mathscr{K} \quad \vec{z} + s = e, \tag{11}$$

$$\tilde{S}\tilde{F}e = 0, \tag{12}$$

$$\vec{n} \geqslant 0, \quad s \geqslant 0, \tag{13}$$

$$\tilde{S} = \quad (s_1, s_2, \ldots, s_N), \quad \tilde{F} = \quad (n_1, n_2, \ldots, n_N).$$

$(\cdot)$.

$\{\vec{n}_k, \vec{z}_k, s_k\}$ $\vec{n}_k > 0$ $s_k > 0$. k

fi , - $\|\vec{d} - \mathscr{K}\vec{n}_k\|$ $\|\mathscr{K} \quad \vec{z}_k + s_k - e\|$ $\vec{n}_k s_k$

, .

, . (10) (12). $[\vec{n}_k, \vec{z}_k, s_k]$ $\beta_k \in [0, 1]$,

$[\Delta_{\vec{n}}, \Delta_{\vec{z}}, \Delta_s]$

$$\begin{bmatrix} \mathscr{K} & 0 & 0 \\ 0 & \mathscr{K} & I \\ \tilde{S}_k & 0 & \tilde{F}_k \end{bmatrix} \begin{bmatrix} \Delta_{\vec{n}} \\ \Delta_{\vec{z}} \\ \Delta_s \end{bmatrix} = \begin{bmatrix} \vec{d} - \mathscr{K}\vec{n}_k \\ e - \mathscr{K}\ \vec{z}_k - s_k \\ \beta_k \mu_k e - \tilde{S}_k \tilde{F}_k e \end{bmatrix}, \tag{14}$$

$\beta_k = (1/N)\vec{n}_k s_k$. τ_k $\vec{n}$ s,

$$\vec{n}_{k+1} := \vec{n}_k + \tau_k \Delta_{\vec{n}}, \quad \vec{z}_{k+1} := \vec{z}_k + \tau_k \Delta_{\vec{z}}, \quad s_{k+1} := s_k + \tau_k \Delta_s. \tag{15}$$

$\vec{n}$ $\vec{n}_0$, *a priori* , .

4.2. Damped Gauss–Newton method

$R(\vec{n}) = \mathscr{K}\vec{n} - \vec{d}$, l^2

$$J[\vec{n}] = \|R(\vec{n})\|_{l^2}^2. \tag{16}$$

H $J[\vec{n}]$, ,

$$g(\vec{n}) = \mathscr{K}\ (\mathscr{K}\vec{n} - \vec{d}), \quad H = \mathscr{K}\ \mathscr{K}. \tag{17}$$

$s_k = \vec{n}_{k+1} - \vec{n}_k$

$$\tfrac{1}{2}\|R(\vec{n}_k) + R'(\vec{n}_k)s\|^2, \tag{18}$$

$$s_k = -R'(\vec{n}_k)\ g_k = -H^{-1} g_k, \tag{19}$$

$$\vec{n}_{k+1} = \vec{n}_k + s_k, \tag{20}$$

$g_k = g(\vec{n}_k)$.

, ..,

$$s_k = -\gamma_k H^{-1} g_k, \tag{21}$$

γ_k -

$$\gamma_k = \quad {}_{\gamma}\ \phi(\gamma) := J[\vec{n}_k + \gamma s_k]. \tag{22}$$

, ( ). H , fi H - fi ; , -.

4.3. Regularization by incorporating an efficient a priori information

s_k

$$s_k = -\gamma_k (H + \alpha_k L)^{-1} g_k, \tag{23}$$

L , α_k . H , . *a priori* -

$$s_k = -\gamma_k (H + \alpha_k L)^{-1} (g_k + \alpha_k(\vec{n}_k - \vec{n}_0)), \tag{24}$$

$\vec{n}_0$ - *a priori* $\vec{n}$. (1994).

4.4. Aerosol particle size distribution function retrieval

$n(r)$, . (7) (24) [0.1, 10] μ. ($N \leqslant 20$) ; , , $N = 200$. , , H , (1978), , $h(r)$ $f(f) : n(r) = h(r)f(r)$, $h(r)$ r, $f(r)$

$$\tau(\lambda) = \int_a^b [k(r, \lambda, \eta)h(r)]f(r)\, r, \tag{25}$$

$k(r, \lambda, \eta) = \pi r^2 Q(r, \lambda, \eta)$ $k(r, \lambda, \eta)h(r)$ Ξ:

$$(\Xi f)(r) = \tau(\lambda). \tag{26}$$

Ξ $\mathscr{K}$. α , α α α . , $\mathscr{K}$. : *a priori* *a posteriori* . *a priori* , α (0, 1). δ , *a posteriori* . (2006). , α :

$$\alpha_k = \alpha_0 \cdot \xi^{k-1}, \tag{27}$$

$\alpha_0 \in (0, 1)$, , $\alpha_0 = 0.1$; $\xi \in (0, 1)$ k k , , α_k k fi .

L

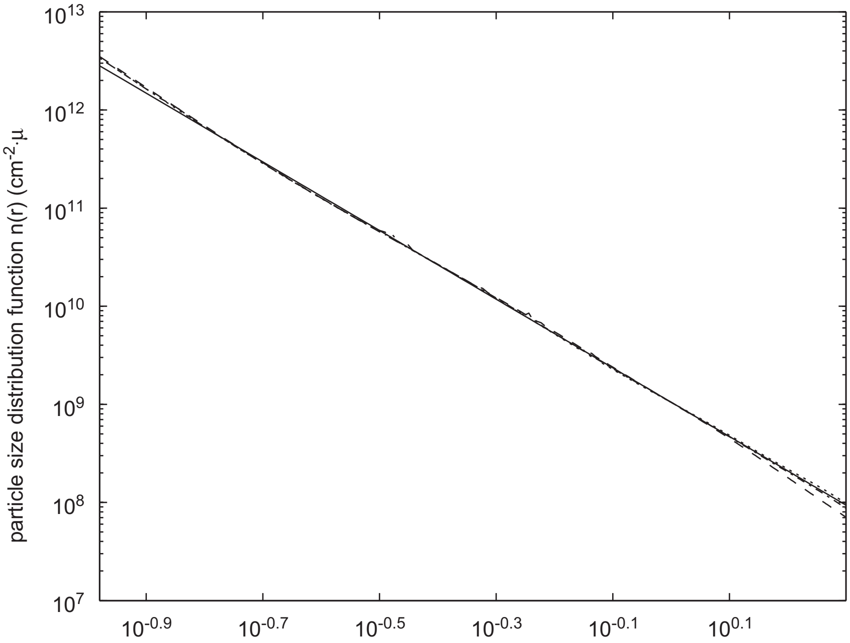
$$L = \begin{bmatrix} 1+\frac{1}{h_r^2} & -\frac{1}{h_r^2} & 0 & \cdots & 0 \\ -\frac{1}{h_r^2} & 1+\frac{2}{h_r^2} & -\frac{1}{h_r^2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\frac{1}{h_r^2} & 1+\frac{2}{h_r^2} & -\frac{1}{h_r^2} \\ 0 & \cdots & 0 & -\frac{1}{h_r^2} & 1+\frac{1}{h_r^2} \end{bmatrix}, \tag{28}$$

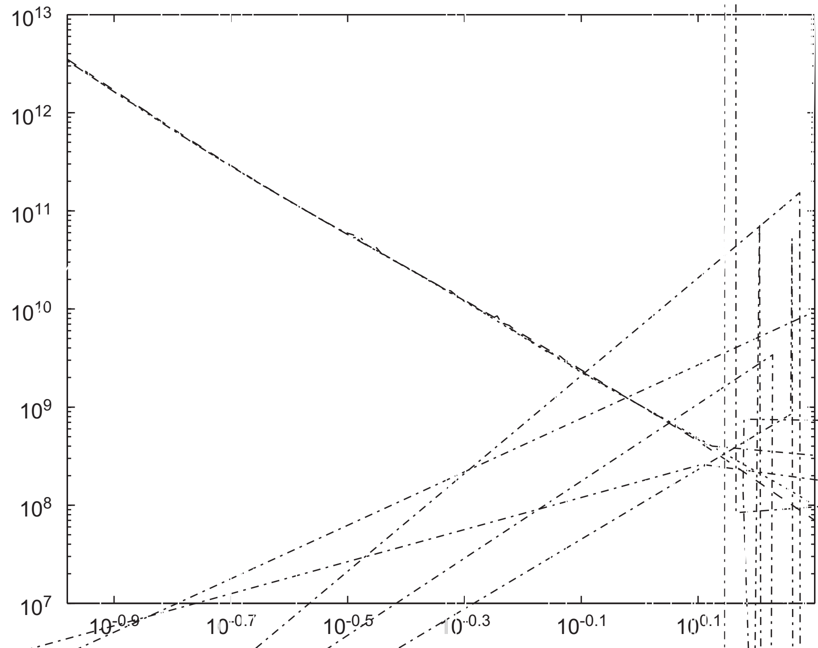
h_r $[a, b]$, $h_r = (b - a)/(N - 1)$, h_r . (., 2006). - , . (24), fi fi $O(\frac{1}{6}N^3)$.

5. Numerical experiments

5.1. Theoretical simulation

, . , ( ) . (3)


particle size distribution function n(r) (cm-2·μ
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^-0.9
10^-0.7
10^-0.5
10^-0.3
10^-0.1
10^0.1


10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^-0.9
10^-0.7
10^-0.5
10^-0.3
10^-0.1
10^0.1
-2.μm-1)

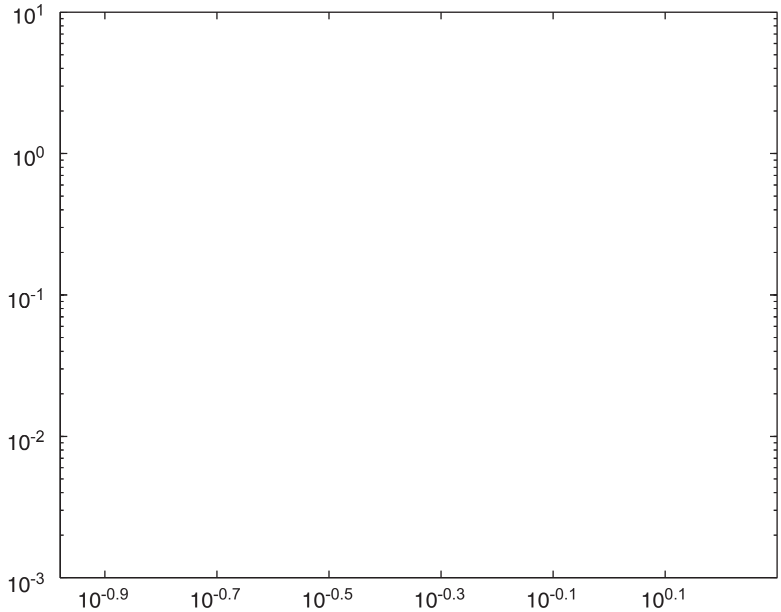
1

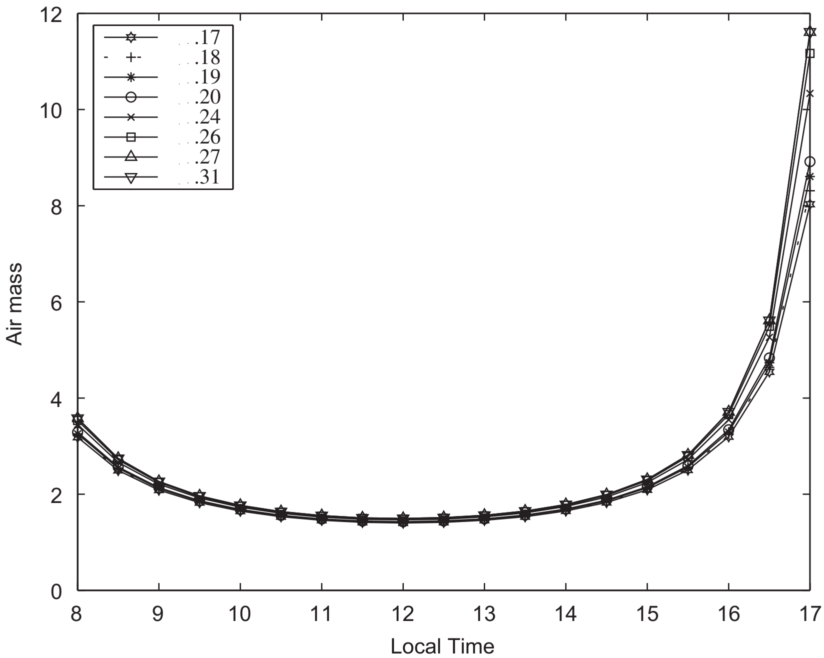
	$\eta = 1.45 - 0.00i$	$\eta = 1.45 - 0.03i$	$\eta = 1.50 - 0.02i$
$\delta = 0.005$	1.6443×10^{-4}	1.2587×10^{-4}	2.2773×10^{-4}
$\delta = 0.01$	1.6493×10^{-4}	1.2720×10^{-4}	2.2847×10^{-4}
$\delta = 0.05$	1.6996×10^{-4}	1.3938×10^{-4}	2.3504×10^{-4}

2

fi *a priori*

	$\eta = 1.45 - 0.00i$	$\eta = 1.45 - 0.03i$	$\eta = 1.50 - 0.02i$
$\delta = 0.005$	17	13	16
$\delta = 0.01$	17	13	16
$\delta = 0.05$	17	13	16





. 5. 17 31, 2005.

318 17 31, 2005. ( ),

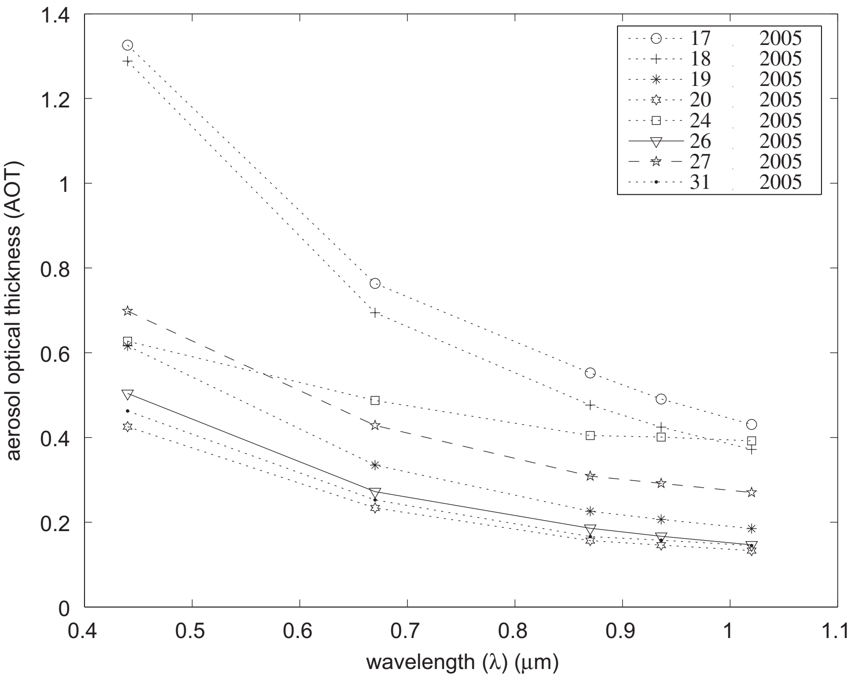
17 20, 24, 26, 27 31 [0.1, 10] μ 17 20, 24 30, 318 ( ),

17 31 . 5. 8:00 16:00 16:00 17:00. λ (μ), 17 20, 24, 26, 27 31. λ . 6.

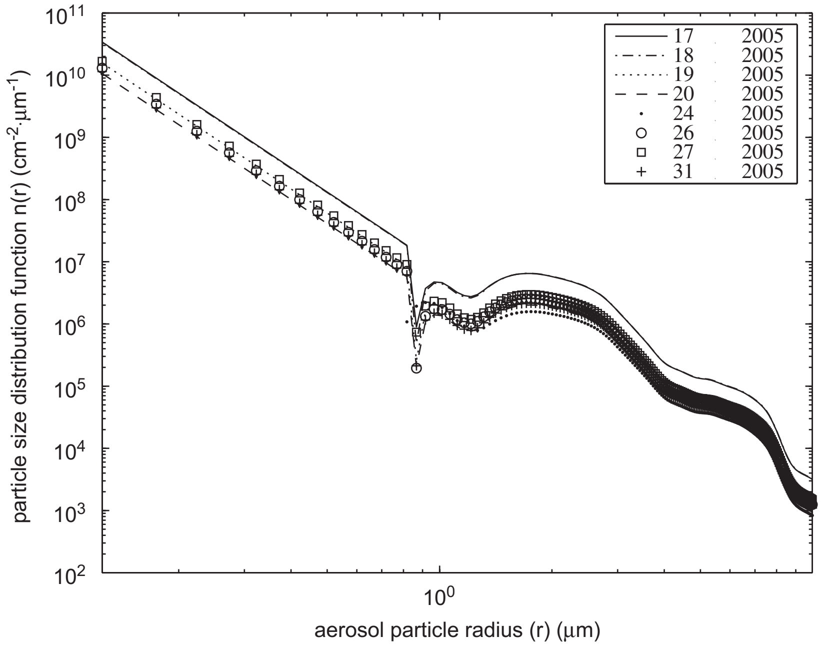
/ fl 17 31,

$\eta = 1.50 - 0.095i$

(1963), H -H (1994), M H (1980). α_0 0.1. α_k 4.4. (& , 1977; ., 2003), α^*, fi α_k fi ( )

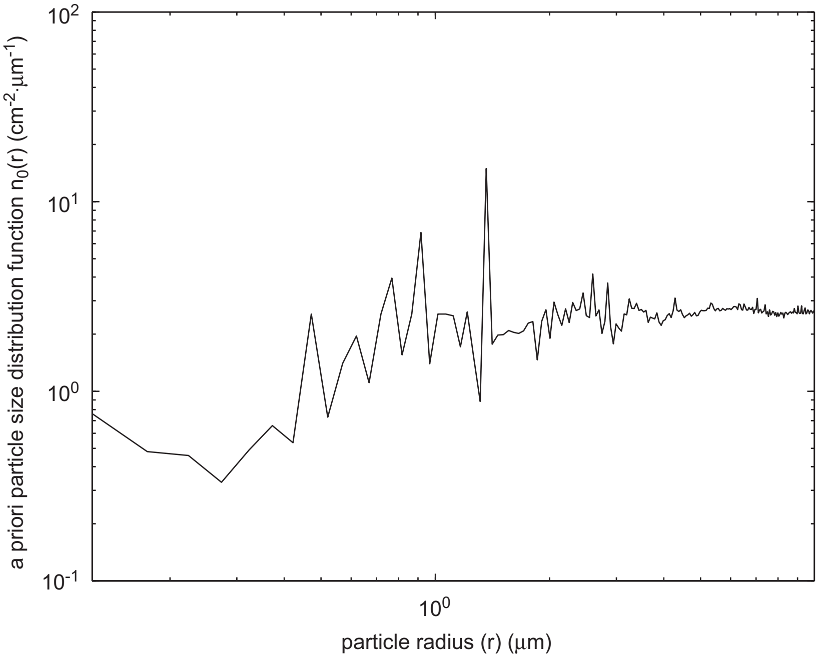


. 6. 17 31 (M), 2005.



. 7. (M), 2005.

. , $n(r)$ 17, 31 . 7 . fi ,



. 8. *A priori* n_0 (M), 2005.

1.0 μ . 1.0 μ , . *a priori* $\vec{n}_0$, . 8 . $\vec{n}_0$ 11. ,

Acknowledgments

H , J ,
fi J , . 10501051
fi H M .

Appendix A. Conception of regularization

3

$$Kn = o, \quad (.1)$$

K H F O.

$$\Pi^{\alpha} : O \rightarrow F, \quad \alpha > 0, \quad (.2)$$

$$\Pi^{\alpha} K n = n \quad n \in F, \quad (.3)$$

$\Pi^{\alpha} K$.
, Π^{α} fi

$$\Pi^{\alpha} = (K^{*}K + \alpha L)^{-1} K^{*}, \quad (.4)$$

L - , K^{*} K, $\alpha \in (0, 1)$.

Appendix B. Conception of *a priori* information

$$\tfrac{1}{2}\|Kn - o\|^2 + \tfrac{1}{2}\alpha \|L^{1/2} n\|^2 \quad (.1)$$

$$n^{\alpha} = \Pi^{\alpha} o. \quad (.2)$$

$$\{\tfrac{1}{2}\|Kn - o\|^2 + \alpha \Omega(n - n_0) : n \in F\}, \quad (.3)$$

n_0 , $\alpha > 0$ Ω .
a priori / Ω n_0,
( )
. (.1).
a priori .

, *a priori* : (1) n_0 l^1 l^1 ; (2) l^2 .

Appendix C. Computing an *a priori* by searching for an interior point solution

a priori n_0 l^1 l^1 . 4.1

$$c\ \vec{n}\quad \mathscr{K}\vec{n}=\vec{d},\ \ \vec{n}\geqslant 0, \tag{.1}$$

$\vec{n}$ N . (.1)

$$\vec{d}\ \vec{z}\quad \mathscr{K}\ \vec{z}+s=c,\ \ s\geqslant 0. \tag{.2}$$

, μ:

$$c\ \vec{n}-\mu\sum_{j=1}^{N}(n_j)\quad \mathscr{K}\vec{n}=\vec{d},\ \ \vec{n}\geqslant 0. \tag{.3}$$

fi

$$_{n_j\to 0}-\mu\ (n_j)=\infty. \tag{.4}$$

$\vec{n}>0$, $\vec{n}>0$. fi

$$L(\vec{n},\vec{z})=c\ \vec{n}-\mu\sum_{j=1}^{N}(n_j)-\vec{z}\ (\mathscr{K}\vec{n}-\vec{d}). \tag{.5}$$

$$\frac{\partial L}{\partial n_j}=c_j-\mu n_j^{-1}-\mathscr{K}_{:j}\vec{z},\quad \frac{\partial L}{\partial \vec{z}_i}=\vec{d}_i-\mathscr{K}_{i:}\vec{n}, \tag{.6}$$

$\mathscr{K}_{:j}$ j $\mathscr{K}$, $\mathscr{K}_{i:}$ i $\mathscr{K}$.

$$_{\vec{n}}L(\vec{n},\vec{z})=c-\mu D^{-1}e-\mathscr{K}\ \vec{z},\quad _{\vec{z}}L(\vec{n},\vec{z})=\vec{d}-\mathscr{K}\vec{n}, \tag{.7}$$

$$D=\begin{bmatrix} n_1 & 0 & \cdots & 0\\ 0 & n_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & n_N\end{bmatrix}. \tag{.8}$$

$$\mathscr{K}\ \vec{z}=c-\mu D^{-1}e,\quad \mathscr{K}\vec{n}=\vec{d},\quad \vec{n}>0. \tag{.9}$$

fi $s=\mu D^{-1}e$,

$$\mathscr{K}\ \vec{z}+s=c,\quad \mathscr{K}\vec{n}=\vec{d},\quad Ds=\mu e,\quad \vec{n}>0. \tag{.10}$$

$(\vec{n},\vec{z},s)$, $\vec{n}\in S_p=\{\vec{n}:\mathscr{K}\vec{n}=\vec{d},\vec{n}>0\}$, $(\vec{z},s)\in S_D=\{(\vec{z},s):\mathscr{K}\ \vec{z}+s=c,s=\mu D^{-1}e>0\}$.

$c \quad \vec{n} - \vec{d} \quad \vec{z} = \vec{n} \quad s = \mu N$, , ,

$$F(\vec{n}, \vec{z}, s) = \begin{bmatrix} \mathscr{K}\vec{n} - \vec{d} \\ \mathscr{K} \quad \vec{z} + s - c \\ Ds - \beta\mu e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (\; .11)$$

β [0, 1].

$(\vec{n}_k, \vec{z}_k, s_k)$, $F(\vec{n}, \vec{z}, s) = 0$

$$F(\vec{n}_k, \vec{z}_k, s_k) \begin{bmatrix} d_{\vec{n}} \\ d_{\vec{z}} \\ d_s \end{bmatrix} = -F(\vec{n}_k, \vec{z}_k, s_k). \qquad (\; .12)$$

$$F(\vec{n}_k, \vec{z}_k, s_k) = \begin{bmatrix} \mathscr{K} & 0 & 0 \\ 0 & \mathscr{K} & I \\ S_k & 0 & D_k \end{bmatrix}, \qquad (\; .13)$$

$$\begin{bmatrix} \mathscr{K} & 0 & 0 \\ 0 & \mathscr{K} & I \\ S_k & 0 & D_k \end{bmatrix} \begin{bmatrix} d_{\vec{n}} \\ d_{\vec{z}} \\ d_s \end{bmatrix} = \begin{bmatrix} \vec{d} - \mathscr{K}\vec{n}_k \\ c - \mathscr{K} \quad \vec{z}_k - s_k \\ -D_k s_k + \beta\mu_k e \end{bmatrix}. \qquad (\; .14)$$

:

Algorithm (*Computing an interior point solution*).

(1) : $(\vec{n}_1, \vec{z}_1, s_1)$ $\vec{n}_1, \vec{z}_1 > 0$, $\varepsilon_{\vec{n}}, \varepsilon_{\vec{z}}, \varepsilon_s > 0$;
(2) : k 1 ∞;
(3)

$$r_{\vec{n}}^k = \vec{d} - \mathscr{K}\vec{n}_k,$$

$$r_{\vec{z}}^k = c - \mathscr{K} \quad \vec{z}_k - s_k,$$

$$\mu_k = \frac{1}{N}\vec{n}_k s_k;$$

(4) $\|r_{\vec{n}}^k\| \leqslant \varepsilon_{\vec{n}}, \|r_{\vec{z}}^k\| \leqslant \varepsilon_{\vec{z}}, \|\mu_k\| \leqslant \varepsilon_s$, ;
(5) $\beta \in (0, 1)$ (.14);
(6)

$$\tau \quad = \quad {}_{\tau \geqslant 0} \left\{ \begin{bmatrix} \vec{n}_k \\ s_k \end{bmatrix} + \tau \begin{bmatrix} d_{\vec{n}} \\ d_s \end{bmatrix} \geqslant 0 \right\}; \qquad (\; .15)$$

(7) $\theta \in (0, 1)$,

$$\tau := \{\theta\tau \quad , 1\};$$

(8)

$$\vec{n}_{k+1} := \vec{n}_k + \tau d_{\vec{n}},$$

$$\vec{z}_{k+1} := \vec{z}_k + \tau d_{\vec{z}},$$

$$s_{k+1} := s_k + \tau d_s.$$

Example (*Computing an interior point solution*).

$$-1n_1 - 2n_2 \qquad n_1 + n_2 = 1, \quad n_1, n_2 \geqslant 0. \qquad (\ .16)$$

$\vec{n} = [n_1 \ \ n_2]$, $c = [-1 \ -2]$, $\mathscr{K} = [1 \ \ 1]$, $\vec{d} = 1$,

$$F(\vec{n}, \vec{z}, s) = \begin{bmatrix} \mathscr{K}\vec{n} - \vec{d} \\ \mathscr{K} \ \ \vec{z} + s - c \\ Ds - \beta\mu e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (\ .17)$$

$\vec{z} = [z_1 \ \ z_2]$, $s = [s_1 \ \ s_2]$, $D = \ (n_1, n_2)$, $\mu = \frac{1}{2}\vec{n} \ \ s$, $e = [1 \ \ 1]$, $\beta = 0.1$ [0, 1].

$$F(\vec{n}_k, \vec{z}_k, s_k) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ s_1 & 0 & 0 & \vec{n}_1 & 0 \\ 0 & s_2 & 0 & 0 & \vec{n}_2 \end{bmatrix}.$$

: $\vec{n}^* = [0.0000 \ \ 1.0000]$, $s \ \ \vec{n}^* = 1.0042e - 007$, $c \ \ \vec{n}^* = -2.0000$.

Appendix D. Damped Gauss–Newton method with line search

$$J[\vec{n}] := \tfrac{1}{2}\|R(\vec{n})\|^2. \qquad (\ .1)$$

$s_k = \vec{n}_{k+1} - \vec{n}_k$

$$\tfrac{1}{2}\|R(\vec{n}_k) + R'(\vec{n}_k)s\|^2, \qquad (\ .2)$$

$s_k = -H^{-1}g_k$ $\vec{n}_{k+1} = \vec{n}_k + s_k$.

(& , 1999), . ., fi γ_k,

$$s_k = -\gamma_k H^{-1} g_k. \qquad (\ .3)$$

γ_k, . γ_k J, , . γ_k fi fi J, :

$$J(\vec{n}_k + \gamma_k s_k) \leqslant J(\vec{n}_k) + c_1\gamma_k g_k \, s_k, \qquad (\ .4)$$

$c_1 \in (0, 1)$.

fi , fi fi γ. , γ_k

$$g(\vec{n}_k + \gamma_k s_k) \ \ s_k \geqslant c_2 g_k \, s_k, \qquad (\ .5)$$

$c_2 \in (c_1, 1)$, c_1 (.4).

(.4) (.5) , $c_1 = 0.1$ $c_2 = 0.4$.

References

[illegible] (1929). [illegible]. *Geografiska Annaler*, *11*, 156 166.

[illegible] (1994). *Ill-posed problems: Theory and applications*. [illegible]

[illegible] (2001). [illegible]. *Applied Optics*, *40*, 1329 1342.

[illegible], & H[illegible] (1983). *Absorption and scattering of light by small particles*. [illegible]

[illegible] (1970). [illegible]. *Journal of Aerosol Science*, *27*, 960 967.

[illegible] (1974). [illegible]. *Journal of Aerosol Science*, *5*, 293 300.

[illegible] (1995). [illegible]. *Applied Optics*, *34*, 5829 5839.

H[illegible] (1994). [illegible]. *Aerosol Science and Technology*, *21*, 46 48.

[illegible] (1963). *Air chemistry and radioactivity*. [illegible]

[illegible] (1978). [illegible]. *Journal of Aerosol Science*, *35*, 2153 2167.

[illegible] (1995). *Vegetation optical remote sensing model and vegetation structural parameterization*. [illegible]

[illegible] (1994). [illegible]. *The Astrophysical Journal*, *425*, 653 667.

[illegible] (1976). *Optics of atmosphere*. [illegible]

[illegible] (1989). [illegible]. *Abstracts of the American association for aerosol research annual meeting* ([illegible]. 330). [illegible]

[illegible] (1999). *Numerical optimization*. [illegible]

[illegible] (1962). [illegible]. *Journal of the Association for Computing Machinery*,