Regularizing active set method for retrieval of the atmospheric aerosol particle size distribution function

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**C e di g a h : f a g_ cf@ ah .c*

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[illegible] S. [illegible] J. G [illegible] P [illegible] 18, 188 (1975); [illegible] F. [illegible] (H [illegible] E [illegible] P [illegible], 2007) [illegible] CE 318 S [illegible] J [illegible] P [illegible] 2008 O [illegible]

010.1100, 010.1110, 280.1100, 100.0100, 100.3190, 000.4430.

1. INTRODUCTION

[illegible] (1 5). [illegible] 1929, [illegible] (AO) [illegible] $\tau(\lambda)=\beta\lambda^{-\alpha}$, [illegible] AO, β [illegible] α [illegible] (6). [illegible]

$$\tau(\lambda) = \int_0^\infty \pi r^2 \, (r,\lambda,\eta) \, (r) \, dr + \varrho(\lambda), \tag{1}$$

[illegible] (e.g., [illegible]), η [illegible] λ [illegible], $\varrho(\lambda)$ [illegible] (r,λ,η) [illegible] M [illegible] S [illegible] AO [illegible] (7,8).

[illegible] (e.g., 9 14), [illegible] (e.g., 15 24), [illegible] (e.g., 25,26), [illegible] (e.g., 27,28) [illegible] (e.g., 29). [illegible] CE 318 [illegible] E . (1). [illegible] S 2 [illegible]

2. ILL-POSED NATURE OF MODEL INVERSION AND REGULARIZATION

A. Ill-Posedness

B. Regularization

3. THEORETICAL DEVELOPMENT

A. Regularizing Active Set Method

C C , * , 1 G + C [ ] l d / C C G .

B C C l , G C l + C C E . (7). : A C C C G 1 G C C C . I G C C (., l d / [ ] + < >).

: G ∈ 1 C C C + C C C C l . S C C . D + C C , l + C G l C l C C C , .

: C C G C ; . C + C G d C C + C .

A C l C d N l G + C C l (7), Cl C C + C G C l C + l G + C G . O C , l C C + C C l (., 35 37). C C C C C / + C l C C C C . S , C C := { ∈ : =0}, + l l = C C

l [+] = 1 2 (,) + (,) + ,

. . . =0, ∈ (8)

= * + ν , = * , C = 1 2 (,) (,). S C C C C C , C + C .

l [ ] = 1 2 (,) + (,),

. . . =0, ∈ . (9)

E . (9) * . N C C ; C C C C + α * C + α. I C C C C * =0. l C d C * . C l G . * . C C + * C C C G , +1 = + * ; C C l C ; . . , +1 = + α * , α / C C

$$\alpha := \mathrm{l}\left\{1, \mathrm{l}\ \frac{\quad}{\in\ ,\ <0}\right\}. \tag{10}$$

I α <1 E . (10), C +1 + C C G . G , C , C C l d + l E . (10) C C C C + C G . C + C * C C l d / + C C + C * . I C / C C =0 C C d C E . (9),

$$\sum_{\in} \lambda^* = {}^* \quad {}^*, \tag{11}$$

l LC C C l + λ * , ∈ * . C C C , + l C C . E . (9) C *

$$ {}^* + \sum_{\in} \lambda^* = 0, \tag{12}$$

$$ {}^* = 0, \quad \in \quad {}^*, \tag{13}$$

$$\lambda^* \geqslant 0, \quad \in \quad {}^*. \tag{14}$$

I l + C G C C / , * C λ * C KC + K + (KK) E . (7) G ∈ , . .,

$$ {}^* \quad {}^* \sum_{\in} \lambda^* = 0, \tag{15}$$

$$ {}^* = 0, \quad \in \quad , \tag{16}$$

$$ {}^* \geqslant 0, \quad \in \quad , \quad \notin \quad , \tag{17}$$

C l + λ * C + C l . I λ * ⩾0 C ∈ , * C l d / ; , l + λ * <0, + C [ ] l C C G ; . ., l C C + l (9) .

BC C C C , + C / C C C + C l C C / + C l C : . 1 ( ).

S 1. G l + C C C 0; 0 C + C C G C 0; + C + C / C C l ν 0 >0 C (l) l C ; := 0 C l + = * + ν .

S 2. S E . (9) . ; I . ≠0, GO O S 3; O , GO O S 4.

S 3. G l + α l E . (10); S +1= +α . ; I α =1, GO O S 5; O , ' ∉ + C ' +α . ' =0 C := ∪{ };

S 4. G l + LC C C l + , λ C C . E . (11); * = ; I λ ⩾0 C ∈ , S OP; + + * = ; O , =C l ∈ λ ; +1= ; := [{ }; GO O S 5;

S 5. S +1:= , := +1 C + C + C / C C C l ν ; GO O S 2.

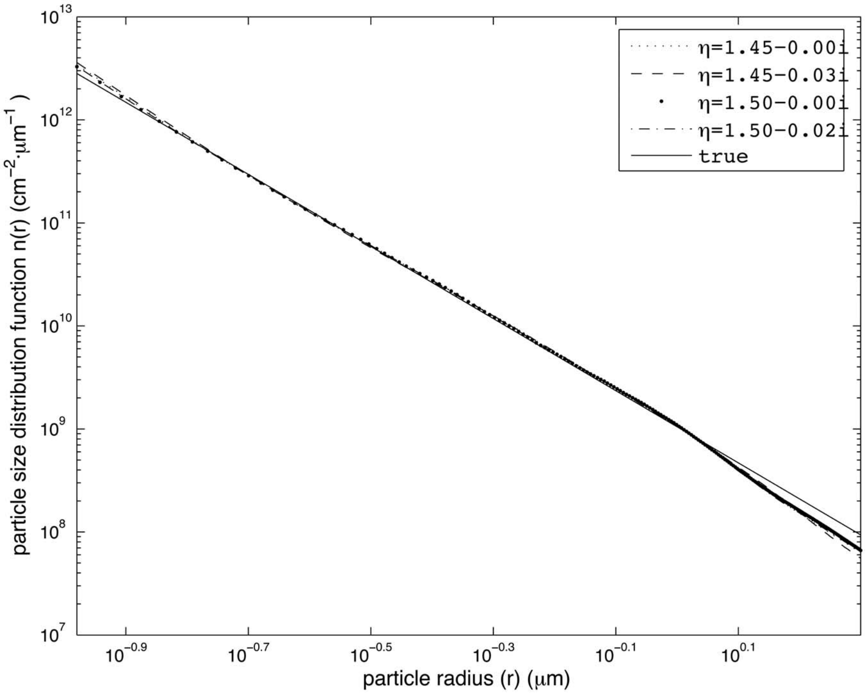
I S 1, l + C = * +ν - C . + E . (9) C C l , C , + C C l (A B), l C C l C . + l C + E . (9) A - B.

. N C + l l + C C + - C / l C C + C [ ], - l C ν; , C + C / C C l C . G l - C C l C -, C (35 37).

B. Choosing the Scale Matrix D and the Regularization Parameter ν

+ C C Cl l l (7) C (9), C C C - C + C / C C Cl ν C C l C . C l C C ; , + 12 C l C C :

$$ = \begin{bmatrix} 1+\frac{1}{2} & \frac{1}{2} & 0 & \cdots & 0 \\ \frac{1}{2} & 1+ & & & \\ & & 21+ & & \end{bmatrix}$$



F . 1. I [illegible] $\delta=0.005$ [illegible]

$$ = + \delta \times (( )), $$

δ [illegible] (0, 1) [illegible]

[illegible]

$$ = \sqrt{\frac{1}{ } \sum_{ =1} \frac{[\tau (\lambda) \quad \tau (\lambda)]^2}{[\tau (\lambda)]^2}}, $$

[illegible] τ [illegible] τ [illegible]

I [illegible] ( ) [illegible] [24]

$$ ( ) = 10.5 \quad ^{3.5} \quad (10^{12} \quad ^2). $$

[illegible] [0.1,2] μm. [illegible] ( ) [illegible] ( ). [illegible] [39,40]

$$ ( ) = \quad ^{(\nu^*+1)}, $$

ν^* [illegible] 2.0 4.0; [illegible] ( ). I [illegible] $\nu^*=3$ [illegible] ( )=10.5 $^{1/2}$ (10 ^{12} ^{2}). [illegible] [15], [illegible] ( )= ^{3} [illegible] ≥0.1 μm. O [illegible]

[illegible]

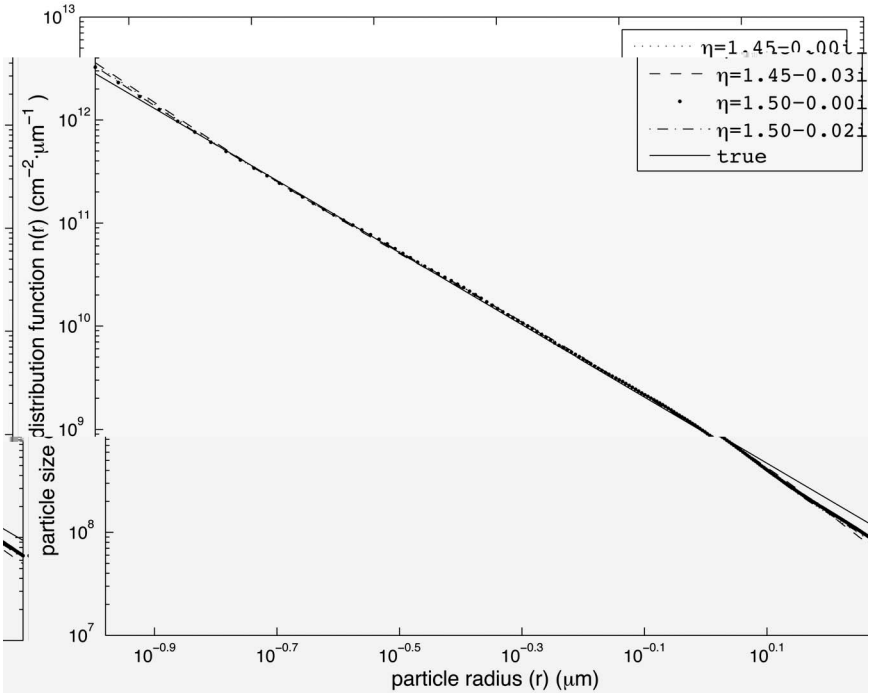
N [illegible] A [illegible] 3.1 [illegible] B.1 (A [illegible] B) [illegible] : [illegible] $\nu_0=0.5$ [illegible] $\xi=0.5$; [illegible] ν [illegible] S [illegible] 3.B; $_0$ [illegible] 0.1; $_0$ [illegible] A [illegible] B; [illegible] =200. I [illegible] =(2 0.1)/(1) [illegible] [,].

F [illegible] η [illegible] 1.45 0.00 . [illegible] η [illegible] η [illegible] 1.45 0.03 , 1.50 0.00 , [illegible] 1.50 0.02 . N [illegible] F . 1 3 [illegible] δ=0.005, 0.01, 0.05 [illegible] 1. I [illegible] 1 [illegible] (10 ^{5}) [illegible] (10 ^{6}), [illegible] [24], [illegible] (10 ^{4}).

O [illegible] [24], [illegible] N [illegible]

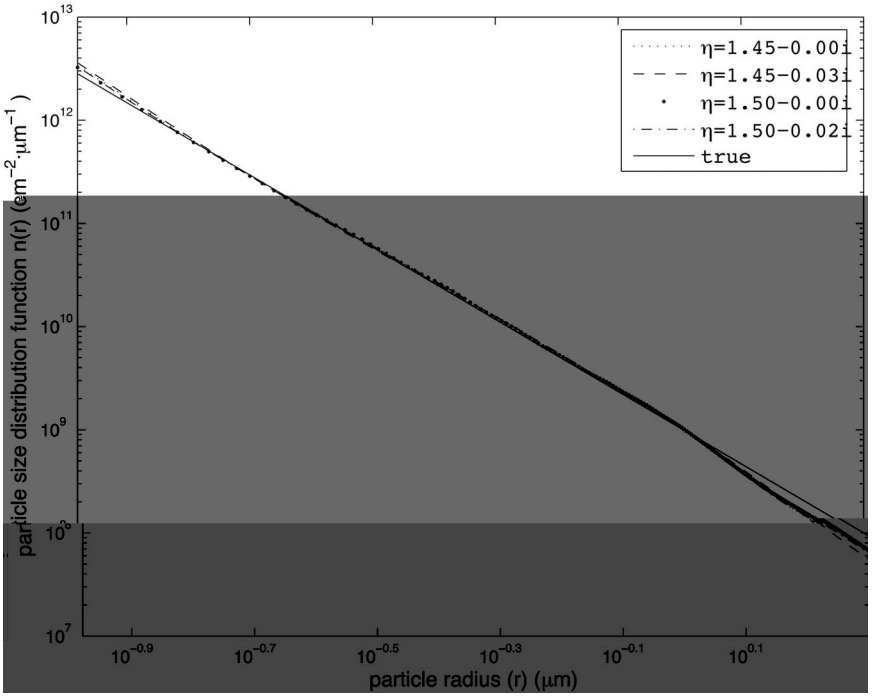
B. Discussion of Numerical Results

I [illegible] CE 318 S [illegible]



F . 2. I C δ=0.01 C

, C 12) C - G C , AO , C C
, C C . C 24 . B C AO
C O 17, 31,
CE 318 O 17, 31, 2005. C A AO O
C CCC C A 17 C 18 C C . H C
C E C S C CERN (C E R -
C N).
I C O 17, 31 C C . C - C C C C . B
C [0.1,10] μ C . F C C - C C C C C C G



F . 3. I C δ=0.05 C

$$(A12)$$

KK ... Eqs. (A10) and (A11) ...

$$\left[\begin{array}{cc} & \\ & 0 \end{array}\right]\left[\begin{array}{c} {}^{*} \\ \lambda^{*} \end{array}\right]=\left[\begin{array}{c} \\ \end{array}\right]. \quad (A13)$$

... ≥ 0 ... $=0$.

APPENDIX B: INSTRUCTIONS ON IMPLEMENTING THE REGULARIZING ACTIVE SET ALGORITHM

[illegible]

$$(B1)$$

... Eq. (7), $=\{ : \geq 0\}$. A ...

$$= 0, \in . \quad (B2)$$

... Eq. (9). ... $+1$ ... $+\alpha$ ...

[illegible] ... (9) ... I ... $\in$ ... $\neq 0$ ... $\alpha \in (0, \alpha)$ ... (1) $+\alpha \in$, (2) $[+\alpha] < [ ]$. ... (2) ... < 0.

[illegible]

... Eq. (9) ... Newton ... [35] ...

Step 1 ... Step 2 ... $\| \| \leq \epsilon$ ... STOP; ...

$$\rho_0 := \ldots := 1.$$

Step 3 ...

$$\alpha := \rho \ldots$$

$$\ldots := \ldots +\alpha \ldots$$

$$\rho := \ldots$$

$$\beta := \rho / \rho$$

Step 4 ... $\| \| \leq \epsilon$ ... STOP; ... $:= +1$, GOTO Step 3.

... 0.1, $\epsilon = 1.0 \times 10^{-8}$ ... 3.1 ... B.1 ...

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