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(: 10501051, 10871191)

(: 2007CB714400, 2005CB422104)

(BRDF) . BRDF

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 , BRDF

,
 BRDF

(Roujean , 1992): $f_{iso}+k_{vol}(t_i, t_v)$ $f_{vol}+k_{geo}(t_i, t_v)$ $f_{geo}=r(t_i, t_v)$, r
 ; k_{vol} k_{geo} ,
 () ; t_i , t_v ; -
 ; f_{iso}, f_{vol} f_{geo} ,
 /

BRDF

BRDF

,
 () ()
). , . Li

(2001) . Pokrovsky

(2002) QR BRDF . Wang (2007)

,
 ,
 (Wang , 2007),
 l^1

(bidirectional reflectance distribution function, BRDF)

[1].

[5, 6], QR BRDF [7],

(NTSVD).

$$y=h(x, S), \tag{1}$$

y , x

S , h , x , S

$$D=h(x, S)+n, \tag{2}$$

$D \in \mathbb{R}^M$, M

M , M

$n \in \mathbb{R}^M$, M

m , $M=m$, (2)

$M > m$, (2) [2].

(Least Squares Solution with Minimum Norm). [3]

“ () ”

[8].

BRDF

[1].

$$f_{iso} k_{vol}(t_i, t_v) f_{vol} k_{geo}(t_i, t_v) f_{geo} r(t_i, t_v), \tag{3}$$

r ; k_{vol} , g_{geo}

[4].

BRDF

. BRDF

(); t_i

; t_v ; -

$f_{iso}, f_{vol}, f_{geo}$,
 BRDF
 RossThick (k_{vol}) LiSparse
 BRDF
 Roujean [11], RossThick,

$$k_{vol}(t_i, t_v) = \frac{1}{\cos t_i \cos t_v} \frac{1 - \cos t_i \sin t_v}{2} \quad (4)$$

$$k_{vol}(t_i, t_v) = \frac{1}{\cos t_i \cos t_v} \frac{\sin t_i \sin t_v \cos t_i}{2} \quad (5)$$

LiSparse

$$k_{geo}(t_i, t_v) = O(t_i, t_v) (\sec t_i \sec t_v) \frac{1}{2} (1 - \cos t_i) \sec t_v \quad (6)$$

$$O(t_i, t_v) = \frac{1}{b} (t_i \sin t_i \cos t_i) (\sec t_i \sec t_v),$$

$$\cos t_i = \frac{h \sqrt{D^2 - (\tan t_i \tan t_v \sin t_i)^2}}{b \sec t_i \sec t_v},$$

$$D = \sqrt{\tan^2 t_i \tan^2 t_v - 2 \tan t_i \tan t_v \cos t_i},$$

$$\cos t_i = \cos t_i \cos t_v \sin t_i \sin t_v \cos t_i,$$

$$\tan t_i = \frac{b}{r} \tan t_v,$$

$$t_i, t_v, h, b, r$$

LiSparse

$$\text{LiSparse} \quad \exp(x) - 1 - x$$

[9],

LiSparse

$$\text{(LiSparseR)}, \quad (6),$$

$$k_{geo}(t_i, t_v) = O(t_i, t_v) (\sec t_i \sec t_v) \frac{1}{2} (1 - \cos t_i) \sec t_i \sec t_v \quad (7)$$

LiSparseR

LiTransit

$$k_{Transit} = \frac{k_{Sparse} B^2}{B k_{Sparse} B^2} \quad (8)$$

$$B: B(t_i, t_v) = O(t_i, t_v) \sec t_i$$

1.2

$$(3)$$

$$Kx=y, \quad (9)$$

$$x = [f_{iso}, f_{vol}, f_{geo}]^T, \quad y = [y_j], \quad y_j$$

$$r_j(t_i, t_v), \quad y$$

BRDF

2

2.1

$$\min J(x), \quad (10)$$

$$\text{s.t. } Kx=y, \quad (11)$$

$$1 \leq c(x) \leq 2, \quad (12)$$

$$J(x) = \sum_{i=1}^n c(x_i), \quad c(x)$$

$$x = [x_1, x_2, \dots, x_n]^T, \quad c(x)$$

$$J(x) = \sum_{i=1}^n c(x_i), \quad c(x)$$

$$J(x) = \sum_{i=1}^n c(x_i), \quad c(x)$$

$c(x)$

Wang [7]

(NTSVD).

[10]

K

$$K = U V^T \begin{matrix} N \\ \vdots \\ i=1 \end{matrix} u_i v_i^T$$

$$U=[u_i] \quad V=[v_i]$$

U

;

K

, K

2.2

/

, i 0

2.2.1

N

[7]

Li [4]

C_p

C_p

(NTSVD),

:

x

$$x^{NTSVD} = \sum_{i=1}^{\tilde{p}} (u_i^T y) v_i, \quad (14)$$

\tilde{p}

[7]

, NTSVD

[11~13]

(10)~(12)

[7]

$$J(x) = 1/2 \|Kx - y\|_2^2 \quad c(x) = x \quad (,)$$

$$J(x) = \|Kx - y\|_2^2 + \|Dx\|_2^2 \quad (13)$$

NTSVD

D

/

D

[7].

$J(x)$

2.2.3 l^1

2.2.2

(NTSVD)

(,)

BRDF

x

?

(LAI), Lambertian

x

Pokrovsky [5] QR

x

“ ” x

$$x^* \quad l^1$$

A.

$$\min_x \|x\|_{l^1}, \quad (15a)$$

2.2.4

$$\text{s.t. } Kx=y, \quad (15b)$$

$$x \geq 0, \quad (15c)$$

l^1

“ ”

(10)~(12)

(15)

[14,15]

(A).

l^1

:

$$S = \{x: Kx = y, x \geq 0\}.$$

S

(15)

BRDF

$$\max y^T g, \quad (16)$$

$$\text{s.t. } s = e - K^T g \geq 0, \quad (17)$$

e

1.

(x, g, s)

$$Kx = y, \quad (18)$$

$$K^T g + s = e, \quad (19)$$

3

$$\tilde{S}\tilde{F}e \geq 0, \quad (20)$$

$$x \geq 0, s \geq 0, \quad (21)$$

73

[4]

73

BRDF

18

BRDF

$$\tilde{S} = \text{diag}(s_1, s_2, \dots, s_N),$$

$$\tilde{F} = \text{diag}(x_1, x_2, \dots, x_N),$$

s_i, x_i

s x

diag(·)

$$\{x_k, g_k, s_k\}, \quad x_k > 0$$

$s_k > 0.$

k

$$\|y - Kx\| = \|K^T g_k + s_k - e\|$$

$$x_k^T s_k$$

1

()

NTSVD

1	LAI
kimes.irrwheat	4
kimes.hardwood	4.2
kimes.soy	4.6
kimes.corn	0.65
kimes.orchgrass	1

4	l^1	WSAs
		(VisRed)
		0.101101803
		0.039901819
		0.050401846
		0.057701894
		0.028101856

2, 3	l^1	NTSVD	(VisRed)
(Nir)			
4	5	2	WSAs

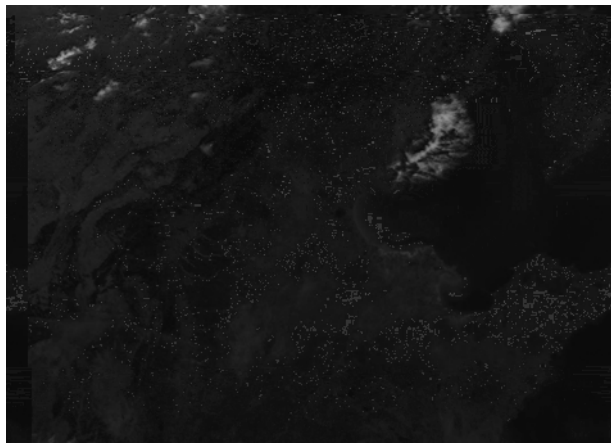
5	l^1	WSAs
		(Nir)
		0.272701813
		0.263901792
		0.497901847
		0.280401851
		0.520901852

2	NTSVD	WSAs
		(VisRed)
		0.060975284
		0.030110122
		0.055554622
		0.108295684
		0.031414520

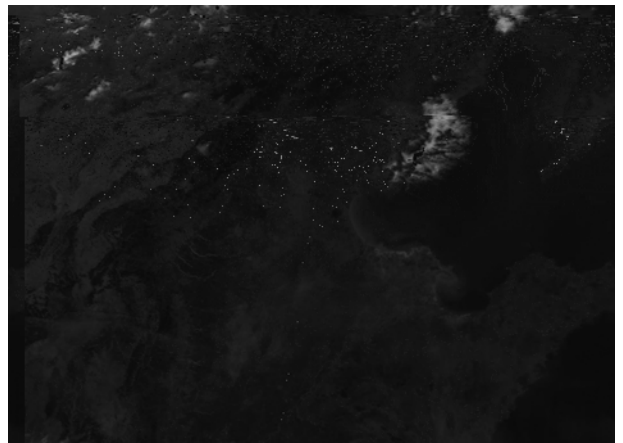
(MODIS)1B BRDF
 MOD021KM.A2001137(26,
 4)
 1B , NTSVD l^1
 3 . 1 DOY=137
 1 BRDF
 MODIS Ambrals
 , MODIS (
 [2] [17]) . MODIS ,
 ,
 BRDF ; ,
 BRDF ,
 ,
 BRDF
 , 2 3 NTSVD
 l^1 . 2 3

	0.060975284	0.077371794
	0.030110122	0.036017748
	0.055554622	0.066419290
	0.108295684	0.078334436
	0.031414520	0.037576732

3	NTSVD	WSAs
		(Nir)
		0.239921484
		0.252075923
		0.364501461
		0.389020678
		0.283347191



1 MOD021KM.A2001137 1 BRDF



3 l^1
MOD021KM.A2001137 1



2 NTSVD
MOD021KM.A2001137 1

4

$$J(x) = \|Kx - y_n\|_2^2 + \|Dx\|_2^2 \quad (22)$$

$$D$$

$$(22) \quad (10)$$

$$K \quad (22) \quad l^p$$

$$(p > 0, \quad p \geq 2), \quad l^p$$

Tikhonov

[7]) (MODIS,

l^1

“ ”

$l^2 \quad l^1$

BRDF

r

$$\|r - r\| \quad \|r\|, \quad 1,$$

(SNR) 1, r

(0,1)

()

[11,18,19]

Tikhonov

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A l^1

$$\min_x e^T x, \text{ s.t. } Kx = y, x \geq 0, \quad (A.1)$$

$$\max_{z,s} y^T z, \text{ s.t. } K^T z \leq e, s \geq 0, \quad (A.2)$$

$$\min_{x_j} e^T x, \text{ s.t. } Kx = y, x \geq 0. \quad (A.3)$$

$$\lim_{x_j \rightarrow 0} \log(x_j) = -\infty. \quad (A.4)$$

$$L(x, z) = e^T x - \sum_{j=1}^N \log(x_j) - z^T (Kx - y), \quad (A.5)$$

$$\frac{\partial L}{\partial x_j} = e_j - \frac{1}{x_j} - K_{j \cdot}^T z, \quad (A.6)$$

$$K_{j \cdot} = K_{j \cdot}^T, K_{i \cdot} = K_{i \cdot}^T.$$

$$\begin{aligned} \text{grad}_x L(x, z) &= e - D^{-1} e - K^T z, \\ \text{grad}_z L(x, z) &= y - Kx, \end{aligned} \quad (A.7)$$

$$D = \begin{bmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_N \end{bmatrix}. \quad (A.8)$$

Karush-Kuhn-Tucker

$$K^T z \leq e - D^{-1} e, Kx = y, x \geq 0. \quad (A.9)$$

$$\begin{aligned} s &= D^{-1} e, \\ K^T z \leq e, Kx = y, Ds = e, x \geq 0. \end{aligned} \quad (A.10)$$

$$\begin{aligned} x &\in S_p = \{x : Kx = y, x \geq 0\}, \\ S_D &= \{(z, s) : K^T z \leq e, s \geq 0\}. \end{aligned} \quad (z, s)$$

$$F(x, z, s) = e^T x - y^T z - x^T s - N, \quad (A.11)$$

$$\begin{aligned} \text{Newton} \\ (x_k, z_k, s_k)^T & \text{ is a sequence such that } F(x_k, z_k, s_k) \rightarrow 0 \end{aligned}$$

$$\text{grad} F(x_k, z_k, s_k) = \begin{bmatrix} d_x \\ d_z \\ d_s \end{bmatrix} F(x_k, z_k, s_k). \quad (A.12)$$

$$\text{grad} F(x_k, z_k, s_k) = \begin{bmatrix} K & 0 & 0 \\ 0 & K^T & I \\ S_k & 0 & D_k \end{bmatrix}, \quad (A.13)$$

$$\begin{bmatrix} 0 & 0 \\ 0 & T \\ k & 0 \end{bmatrix} \begin{matrix} x \\ z \\ s \end{matrix} \begin{matrix} k \\ T \\ k \end{matrix}.$$

A.1 , x_1